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SHAPE AND INTERNAL STRUCTURE OF MOON,
FROM LUNAR ORBITER DATA

for

NATIONAL AERONAUTICS AND SPACE ADMINISTRATION
LUNAR ORBITER PROGRAM OFFICE

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PREFACE

This report presents the results of a study of the shape and internal structure of the Moon utilizing data from the Lunar Orbiter Program under NASA Contract No. NSR 05-264-002. It is planned to submit the report, less appendix, for consideration for publication in ICARUS.

SUMMARY

The distribution of continents and maria and the displacement of the Moon's center of figure from the center of mass indicate that the average elevation of continents is greater than that of maria. Comparison of the terms in a spherical harmonic expansion of the lunar gravity field with similar terms in distributions of highlands (continents and areas not within circular basins) reveals that the gross shape, as reflected in the distribution of highlands, is unrelated to the gravity field. Consideration of the $C_{2,0}$, $C_{2,2}$, and $C_{3,0}$ terms in the gravity field and of the distribution of surface features indicates that excess mass underlies the maria and circular basins. Density variations and internal structures which would produce the observed gravity field are calculated for (1) a rigid Moon with a random lumpiness and (2) a Moon in isostatic equilibrium. The density contrasts and dimensions for each model are equally probable. Orbiter data are insufficient to determine the overall moment of inertia and whether the Moon is homogeneous or differentiated.

ACKNOWLEDGMENTS

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INTRODUCTION

Analyses of tracking data from the U.S. Lunar Orbiter series of spacecraft (Michael, et al., 1967; Tolson and Gapcynski, 1967; Michael, 1967; Lorell and Sjogren, 1968) and the Soviet Luna-10 (Akim, 1966) have yielded values for the coefficients of a spherical harmonic expansion of the external lunar gravity field. Lunar Orbiter photographs of the backside provide Moonwide data on the distribution of continents, maria, and circular basins (Lunar Farside Charts LFC-1 and LFC-2, 1967). Determinations of the lunar radius by image motion study of Lunar Orbiter photographs (Michael, 1967) and radar observations (Shapiro et al., 1967) have yielded additional information on the relationship between the Moon's center of figure and center of mass. Because of this increased knowledge, it is appropriate to reconsider the relationships between the Moon's gravitational field, internal structure, shape, and the distribution of gross surface features discussed previously (Lamar and McGann, 1966a).

RELATIVE ELEVATION OF CONTINENTS AND MARIA

Determinations of the lunar radius by analysis of ranger impacts (Sjogren and Trask, 1965), image motion study of Lunar Orbiter photographs (Michael, 1967), and radar observations (Shapiro, et al., 1967) have yielded values which are inconsistent with the elevations shown on the charts of the Aeronautical Chart and Information Center (ACIC), U.S. Air Force, St. Louis. The origin of coordinates for the charts is a mean sphere which best fits the center of volume, whereas the origin for the other determinations is the center of mass. Thus it has been suggested (Sjogren, 1966; Michael, 1967; Shapiro, et al., 1967) that the discrepancies may be the result of displacement between the Moon's center of figure and center of mass.

The relationship between the center of figure and center of mass must be known to accurately determine the relationships between the gravity field, shape and internal structure. For such studies it will be necessary to prepare contour maps showing elevations with respect to the Moon's center of mass. The Moon's gross shape has been approximated on the assumption of a systematic difference in elevation between continents and maria (Lamar and McGann, 1966a). In their analysis of the relationships between the Earth's shape, gravity field and internal structure, Munk and MacDonald (1960a) followed a similar approach by assuming a systematic difference in elevation between continents and ocean basins.

Lamar and McGann (1966a) suggested that the average elevation (relative to the center of mass) of the continents is 3 km greater than the maria. Goudas (1966) questioned this assumption, pointing out that recent stereoscopic elevation determinations reveal no such relationship. The origin of coordinates for these elevations is a mean sphere which best fits the points or the center of volume. The relationship between the center of volume and center of mass must be known before any systematic difference in elevation between continents and maria can be established from stereoscopic observations.

The problem may be visualized by imagining that the Earth lacks oceans, which provide a convenient level surface centered on the Earth's

center of mass. An observer on the Moon studying the Earth's shape
by stereoscopic methods or observations of the limb would logically
choose the Earth's center of volume as the origin of coordinates. If
our lunar observer viewed the Earth with the center of the Pacific
Basin on one limb, the center of the Pacific Basin would have about
the same elevation as continental areas on the Earth's opposite side,
relative to a coordinate system with its origin at the center of the
Earth's disk or center of volume.

Similarly in the case of the Moon, it is possible that the con-
tinents are systematically higher relative to the center of mass and
that some maria surfaces, relative to the center of figure, are higher
than some continental areas. Therefore, it will not be possible to
use the stereoscopic height determinations to establish any systematic
Moonwide difference in elevation between continents and maria until
such observations are transformed so that the origin of coordinates
is the center of mass. The authors (Lamar and McGann, 1966a,b) were
thus incorrect in stating that Hedervari's analysis of Baldwin's (1961,
1963) data was pertinent, and the relative accuracy of stereoscopic
determinations by different investigators discussed by Goudas (1966)
has no bearing on the problem.

As pointed out by O'Keefe and Cameron (1962) prior to the Ranger
and Lunar Orbiter programs, the Moon's center of disk was known to be
situated about 1 km south of the center of mass. O'Keefe and Cameron
made a least squares solution to determine the sphere which most nearly
fits the coordinates determined from stereoscopic observations cataloged
by Schrutka-Rechtenstamm (1958). The origin of Schrutka-Rechtenstamm's
coordinates is a center of disk derived from limb observations.

O'Keefe and Cameron found that the center of the mean sphere has
the following coordinates with respect to the origin: $y = -0.4 \pm 0.4$ km
and $z = -1.4 \pm 0.4$ km where the x axis points to the Earth, y points
east, and z is the north polar axis. Their analysis indicates that
the center of the mean sphere, fitting the stereoscopic observations,
should lie about 2 km south of the center of mass. This displacement
of the center of figure from the center of mass is consistent with the
higher proportion of continental areas in the southern hemisphere

compared to the northern hemisphere (Fig. 1 and 2) and with a systematic excess in elevation of continents over maria. Although the east-west separation is small and uncertain, the apparent direction of separation is opposite of what would be expected from the greater proportion of continental areas in the western hemisphere and from an excess in elevation of continents over maria.

Analysis of tracking data from the Ranger flights to the Moon (Sjogren and Trask, 1965) and the first photographs (Lipskii, 1961) of the Moon's farside provided the first indication that a relationship analogous to that between the north and south hemispheres exists between the Moon's farside and Earth-facing hemispheres. The farside pictures indicate that there is a much smaller percentage of maria on the farside than on the hemisphere facing the Earth. If a systematic difference in elevation between continents and maria exists, the Moon's center of figure relative to the center of mass should be displaced away from the Earth. Tracking of the Ranger spacecraft revealed that the radius from the center of mass is about 3 km less than the value indicated on the ACIC charts (Sjogren and Trask, 1965). Since the origin of coordinates for these charts is based on a mean sphere determined from stereoscopic observations, the Ranger data indicate that the center of figure is displaced away from the Earth. This displacement could have been predicted from the relative absence of maria on the farside and the assumption of a systematic excess in elevation of the continents over maria.

Preliminary determinations of the Moon's radius with respect to the center of mass in the equatorial region facing the Earth have been accomplished by analysis of image motion on pictures taken by Lunar Orbiter (Michael, 1967). This investigation also revealed that the radii are systematically lower by 1 to 3 km (average about 2 km) than the radii on the curve obtained from harmonic analysis of the ACIC selenodetic control system by Bray and Goudas (1966). Measurements of the lunar radius, made by combining radar determinations of the distance to sub-earth points on the Moon, and range data of Lunar Orbiter 1 and 2 also produced values about 1 to 2 km less than the radii determined from the stereoscopic observations (Shapiro, et al., 1967). Thus the
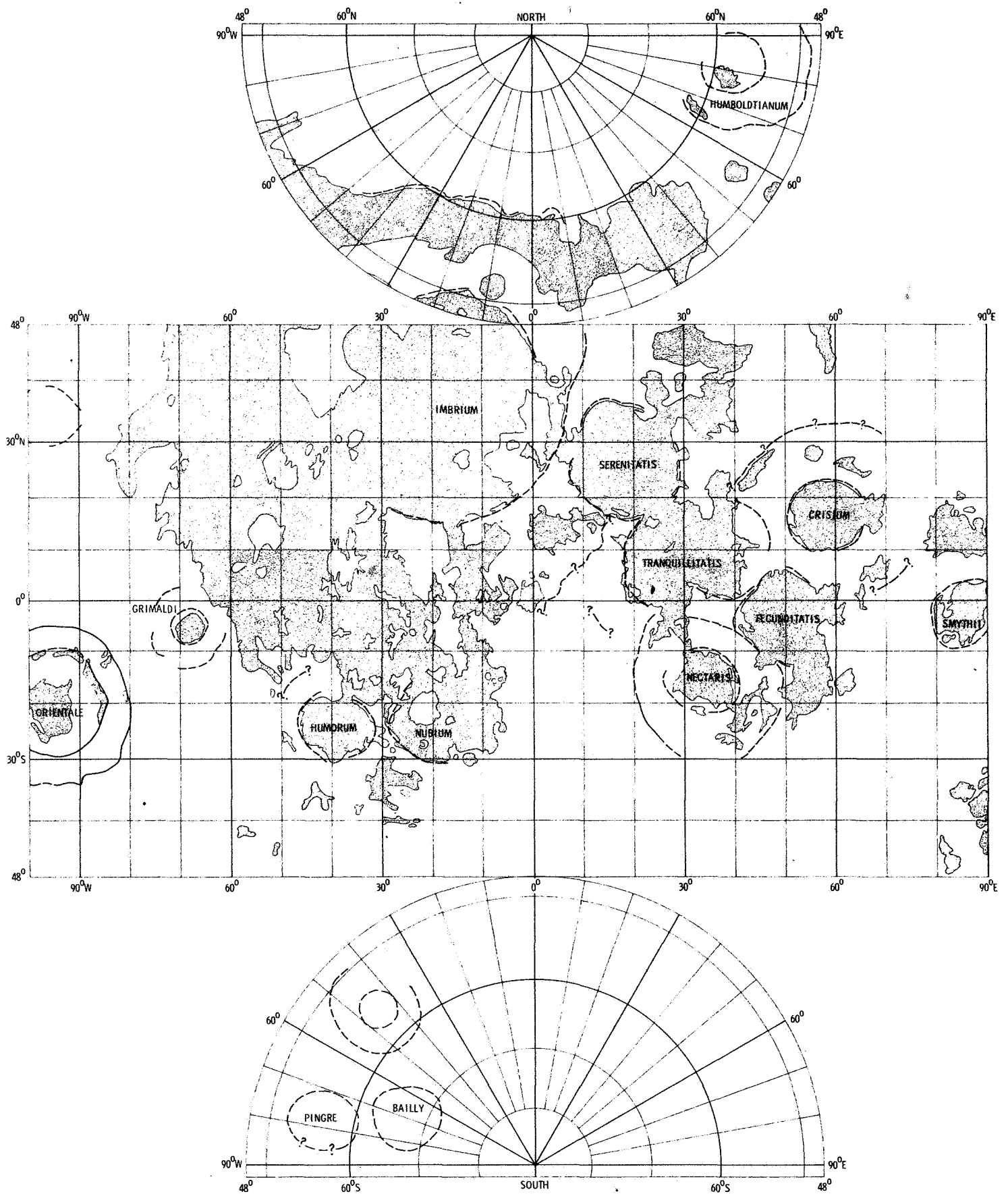
Fig. 1 Earth-facing side of the Moon showing distribution of continents, mare, and circular basins.
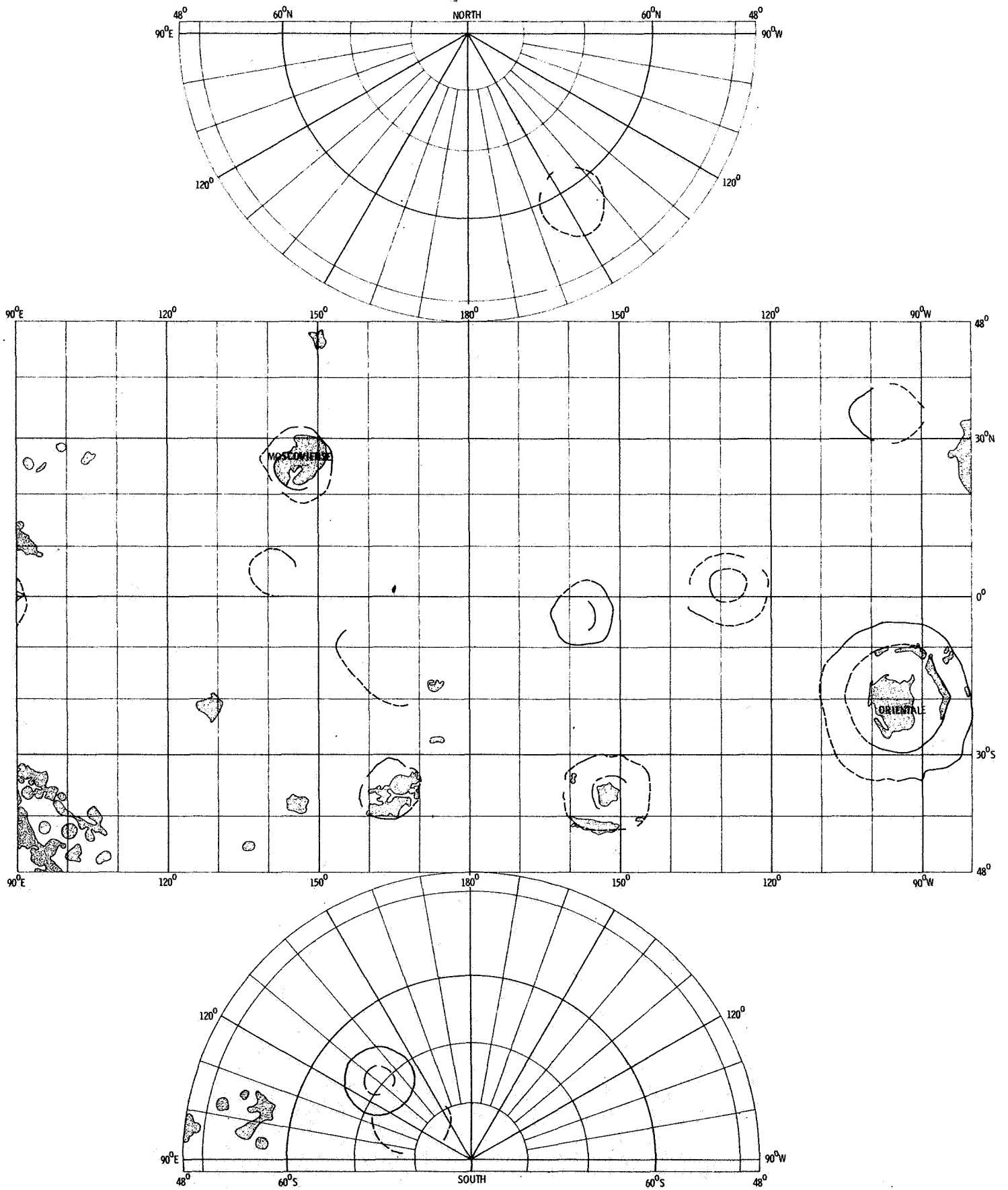
Fig. 2 Farside of the Moon showing distribution of continents, mare, and circular basins.

analysis of data from the Lunar Orbiters substantiates the hypothesis of a systematic excess of elevation of continental areas over maria, which is related to a displacement of the center of figure from the center of mass.

As shown in Fig. 3, if a systematic difference in elevation, amounting to H, exists between continents and maria on opposite hemispheres, then the displacement between the center of mass and center of disk is H/2. The displacement of the center of mass 1 km north of the center of disk on the Earth-facing hemisphere is consistent with the higher percentage of continental areas in the Moon's southern hemisphere and with a systematic difference in elevation of about 3 km.

The displacement of about 2 km between the Earthward and farside hemispheres (corresponding to H/2 on Fig. 3) leads to an unexpectedly high estimate of about 5 km for the excess in elevation of continents over maria (H on Fig. 3). However, the radii determinations by image motion are concentrated in the equatorial region (Michael, 1967), which is predominantly maria; thus the relative percentage of maria on the Earthward side may be overemphasized. The existing data appear to establish that continents are systematically higher than maria relative to the Moon's center of mass. Additional studies are required to determine the magnitude of this difference and to learn the average depth of the circular basins.

As previously noted (Lamar and McGann, 1966a), if the Moon's density distribution is spherically symmetrical, then to a first approximation the orientation of the axes of the principal moments of inertia would be related to distribution of continents, low-lying maria, and circular basins. Low-lying areas would be concentrated in polar regions and around an axis in the equatorial plane at 90°-270° east longitude.
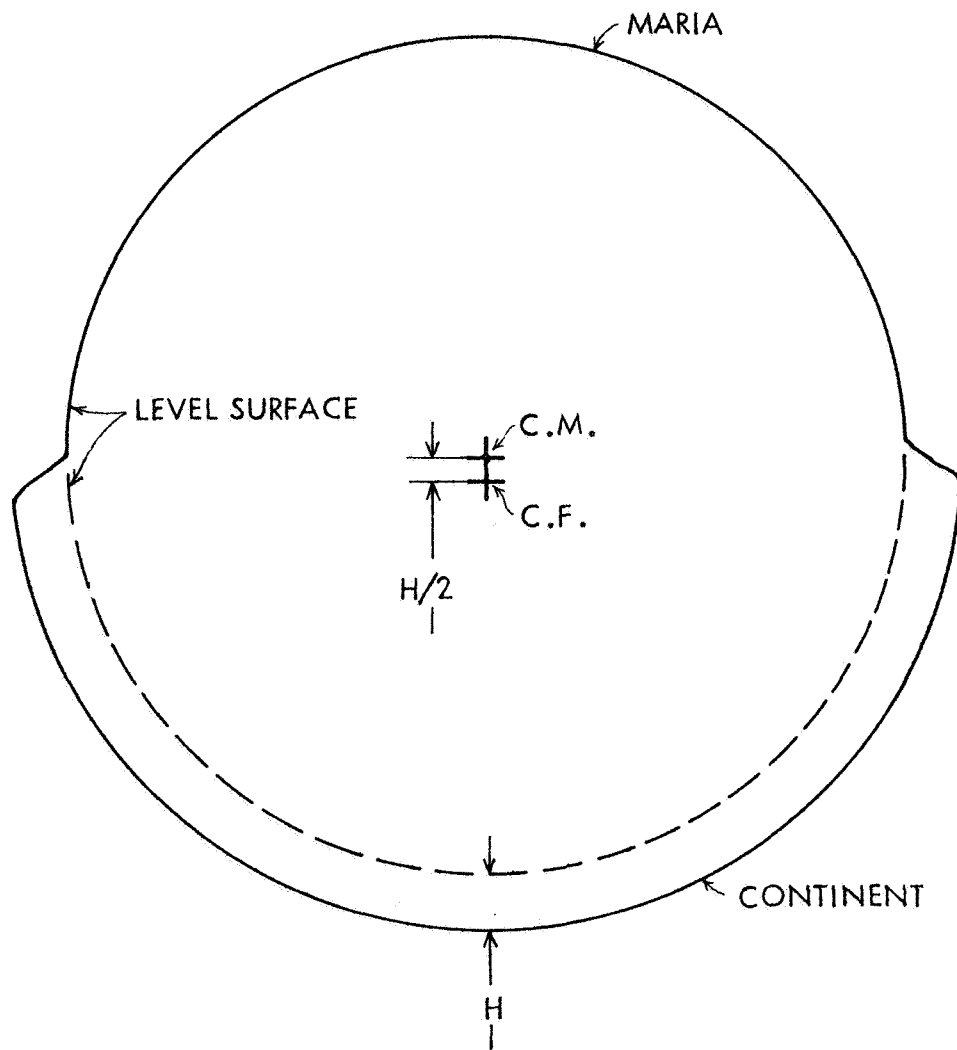
Fig. 3 Relationship between center of mass (c.m.) and center of figure
(c.f.) for concentration of maria and continents in distinct
hemispheres. A systematic excess in elevation (H) of continents
over maria with respect to a level surface is assumed.

DISTRIBUTION OF CONTINENTS, MARIA, AND CIRCULAR BASINS

Figures 1 and 2 show the Moonwide distribution of continents,
maria, and circular basins greater than 300 km in diameter. The maps
were compiled from the USAF Lunar Reference Mosaic (1960), the USAF
Project Apollo Lunar Planning Chart (1962), Whitaker, et al. (1963),
Lunar Farside Chart LFC-1 (1967), and by study of Lunar Orbiter photo-
graphs. These maps indicate that the Moonwide distribution of low-
lying maria and circular basins is the opposite of that required to
explain the orientation of the axes of principal moments of inertia.
That is, low-lying maria and circular basins appear to be concentrated
in equatorial regions and around an axis in the equatorial plane at
$0°-180°$ east longitude.

In order to determine the orientation of the principal axes of
the distribution of gross surface features and models of internal struc-
ture, it is convenient to express the distribution of surface features
in terms of the coefficients of spherical harmonic representations of
functions analogous to the ocean function of the Earth presented by
Munk and MacDonald (1960b). A computer program utilizing the equations
presented by Munk and MacDonald was used to calculate coefficients of
spherical harmonic expansions for distributions of gross surface fea-
tures. Because the expansion used by Munk and MacDonald is different
from the expansion of the gravity field in spherical harmonics given
below, each coefficient in the functions was multiplied by $(n-m)!/n!$
to permit direct comparison with similar terms in the gravity field.

In calculating the coefficients, the proportion of the surface
feature considered was tabulated for squares 10 degrees on each side
from Fig. 1 and 2. The values of the coefficients are listed on Table
I. Distribution 1 represents a continentality (or non-maria) function.
The coefficients of Distribution 2 express the distribution of areas not
within the outer rim of a circular basin. In defining the area within
a circular basin, questioned outer rims were ignored and incomplete
rings were completed.

Muller and Sjogren (1968) have shown that the most youthful ap-
pearing circular basins on the nearside (Imbrium, Serenitatis, Crisium,

n	m	$C'_{n,m}$				$S'_{n,m}$			
		Dist. 1	Dist. 2	Dist. 3	Dist. 4	Dist. 1	Dist. 2	Dist. 3	Dist. 4
2	0	0.146	0.0841	0.0442	0.0259	-	-	-	-
	1	-0.0994	-0.0852	-0.0753	-0.0322	0.0309	-0.0238	-0.00882	-0.0157
	2	-0.0228	-0.0233	-0.0133	-0.00581	0.0282	-0.0178	-0.000306	-0.00612
3	0	0.104	0.0179	0.0362	0.0257	-	-	-	-
	1	0.0274	-0.0178	-0.0370	-0.00749	0.00335	0.0196	0.00428	0.000342
	2	-0.00943	-0.0171	-0.0235	-0.0108	0.00768	0.00661	0.0110	-0.000499
	3	0.00599	0.00124	-0.00192	-0.000845	0.00382	-0.00697	-0.00399	-0.00185
4	0	-0.00377	-0.0256	0.00303	0.0101	-	-	-	-
	1	0.0129	0.00326	-0.00265	0.00757	-0.00267	0.0152	0.0218	0.0177
	2	-0.00283	-0.00477	-0.00898	-0.00407	0.00335	0.0131	0.0108	0.00330
	3	0.00078	-0.00216	-0.00195	-0.000917	-0.00043	0.00253	0.00327	0.000544
	4	0.00098	0.000290	0.000080	0.0000092	0.00023	0.000263	0.000084	0.000098
5	0	0.0125	0.140	0.0936	0.0254	-	-	-	-
	1	0.00298	0.0241	0.0295	0.0196	-0.0143	-0.0280	-0.000272	-0.00302
	2	-0.00195	0.000388	0.000342	0.00106	-0.000021	-0.000734	-0.00152	0.00101
	3	-0.00050	-0.00105	-0.000645	-0.000502	0.00016	0.00183	0.000930	0.000292
	4	0.00011	0.000027	0.000012	-0.000030	-0.00023	0.000124	0.000240	0.000050
	5	0.000068	0.000080	0.000023	0.000016	-0.0000078	0.000030	0.000034	0.000020
6	0	-0.0415	-0.0206	-0.0671	-0.0920	-	-	-	-
7	0	-0.00167	-0.0550	-0.0804	-0.0488	-	-	-	-
8	0	-0.0423	0.00485	0.00861	-0.00886	-	-	-	-

Table I - Coefficients of spherical harmonic representations of surface features.
Nature of distributions given in text.

Humorum, and Nectaris) and Orientale Basin on the limb caused perturba-
tions in the motions of Lunar Orbiters. They suggest the perturbations
are due to excess mass beneath these basins. Comparison of the outlines
of the gravity anomalies shown on their map with Fig. 1 indicates that
the excess masses are concentrated within the inner rings of the Imbrium
and Nectaris Basins. For the Crisium and Humorum Basins, the existence
of an outer rim or ring relative to the excess mass concentration is
not certain. In the case of the Serenitatis Basin, the existence of
an outer ring is not clear. A suggestion of an outer rim appears
along the northeast shore of Lacus Somniorum and the south shore of
Mare Vaporum and the area to the southeast along the Hyginus Rille.

Desiree Stuart-Alexander and Keith Howard (personal communications,
1968) have made a preliminary study of the relative ages of circular
basins greater than 300 km in diameter on the basis of (a) ratio of
basin size to size of larger superposed craters, (b) sharpness of scarp,
(c) presence of rim deposits, (d) completeness of outer ring, (e) cir-
cularity, (f) completeness of an inner ring, and (g) subjective rank
of overall age. Based on their analysis, Serenitatis is the oldest
of the nearside basins which perturbs the motions of the Lunar Orbiters.
They consider the following additional basins to be younger than Seren-
itatis: Grimaldi, Bailly, Moscoviense, and unnamed basins identified
by the coordinates of their centers, 45W, 55S; 140E, 5N; 165E, 35S;
130E, 70S; 155W, 5S; and 130W, 5N. Distribution 3 represents the areas
outside of the outer rims of circular basins as young as Serenitatis.

In considering the significance of the inner and outer rings or
rims, it is important to note that the development of the inner ring
appears to be dependent on the diameter of the outer ring. For example,
in the following small basins younger than Serenitatis, the inner ring
is absent: Bailly; 140E, 5N; and 165E, 35S. For the following slightly
larger basins younger than Serenitatis, the inner ring is indistinct
or incomplete compared to the outer rim: Moscoviense; 130E, 70S; and
155W, 5S. Therefore, in considering the distribution of areas outside
of inner rings, single-ringed basins were ignored. Distribution 4
represents areas outside of the inner rings of the basins which Muller

and Sjogren (1968) suggest are underlain by excess mass, and the following additional basins: Grimaldi; Moscoviense; 45W, 55S; 130E, 70S; 155W, 5S; 130W, 5N.

Because of variations in illumination, the tone of flat lying areas differs with different photos and the precise distribution of maria is difficult to determine. The outlines of the circular basins are also commonly obscured. Although the distribution of gross surface features shown on Fig. 1 and 2 may be modified by more detailed studies, the values of the coefficients in Table I should not change significantly with subsequent analysis of improved maps.

LUNAR GRAVITY FIELD

The lunar gravitational potential function is expressed as the following series expansion in spherical harmonics with coefficients $C_{n,m}$ and $S_{n,m}$:

$$U = \frac{\mu}{r} \left[1 + \sum_{n=2}^{\infty} \sum_{m=0}^{n} \frac{a}{r}^n P_{n,m}(\sin \phi)(C_{n,m} \cos m\lambda + S_{n,m} \sin m\lambda) \right]$$

where μ is the product of the gravitational constant and the mass of the moon, r is the radial distance from the center of the moon, a is the mean radius of the moon, $P_{n,m}$ are associated Legendre polynomials, ϕ is the latitude, and λ is the longitude. This is the form of the potential recommended by the International Astronomical Union and has been adopted in analyses of the Lunar Orbiter tracking data (Michael, et al., 1967; Tolson and Gapcynski, 1967; Michael, 1967; Lorell and Sjogren, 1968). The values of the coefficients in the gravity field presented by Lorell and Sjogren (1968), Tolson and Gapcynski (1967), and Akim (1966) with the correction in sign of the terms with odd value of m, suggested by Tolson and Gapcynski (1967), are presented on Table II.

It is interesting to note that in all three analyses the $C_{2,1}$ term departs significantly from zero. In Lorell and Sjogren's (1968) analysis, the $S_{2,2}$ term also departs from zero. Jeffreys (1962, p. 141) pointed out that the lack of a wobble of the Moon's axis of rotation indicates that the $C_{2,1}$ and $S_{2,1}$ terms in the gravity field must be very nearly zero or no more than 0.001 of the $C_{2,2}$ and $S_{2,2}$ terms. If the $S_{2,2}$ term is not very close to zero, then the resulting torque should cause the Moon to rotate until the line pointing toward the Earth is a principal axis and this term vanishes. According to Michael (1967), the $C_{2,1}$, $S_{2,1}$, and $S_{2,2}$ coefficients have tended to decrease as additional tracking data have been analyzed. It is possible that, with sufficient data and further analyses, these terms will completely vanish.

n	m	$C_{n,m} \times 10^{-4}$ Akim (1966)	$C_{n,m} \times 10^{-4}$ Tolson and Gapcynski (1967)	$C_{n,m} \times 10^{-4}$ Lorell and Sjogren (1968)	$S_{n,m} \times 10^{-4}$ Akim (1966)	$S_{n,m} \times 10^{-4}$ Tolson and Gapcynski (1967)	$S_{n,m} \times 10^{-4}$ Lorell and Sjogren (1968)
2	0	-2.06 ± .22	-2.0596 ± .141	-2.0263 ± .0143	—	—	—
	1	-.157 ± .059	-.1661 ± .051	-.0878 ± .0131	-.0361 ± .0358	.0080 ± .039	.0150 ± .0139
	2	.140 ± .012	.2042 ± .029	.2191 ± .0249	-.0139 ± .0145	-.0342 ± .025	.1310 ± .0335
3	0	-.363 ± .099	-.3773 ± .180	-.2223 ± .0262	—	—	—
	1	.568 ± .026	.3012 ± .048	.3636 ± .0025	.178 ± .032	.1762 ± .053	.0740 ± .0032
	2	.118 ± .047	.1294 ± .028	-.0257 ± .0058	-.00702 ± .046	-.0147 ± .033	-.0200 ± .0063
	3	—	.0317 ± .015	-.0265 ± .0079	—	-.0043 ± .018	-.0496 ± .0114
4	0	.333 ± .270	.0798 ± .128	.0941 ± .0190	—	—	—
	1	—	-.1560 ± .036	-.1236 ± .0046	—	.0391 ± .028	.0564 ± .0051
	2	—	.0011 ± .010	.0361 ± .0034	—	.0072 ± .013	.0051 ± .0035
	3	—	-.0082 ± .008	.0164 ± .0021	—	-.0001 ± .006	-.0276 ± .0015
	4	—	-.0007 ± .003	.0091 ± .0011	—	.0011 ± .003	.0079 ± .0011
5	0	—	-.5505 ± .171	-.1614 ± .0321	—	—	—
	1	—	-.0385 ± .037	—	—	.0829 ± .031	—
	2	—	.0342 ± .009	—	—	-.0203 ± .008	—
	3	—	-.0071 ± .002	—	—	-.0078 ± .002	—
	4	—	-.0008 ± .001	—	—	-.0013 ± .001	—
	5	—	-.0003 ± .0002	—	—	.0003 ± .0002	—
6	0	—	—	-.1089 ± .0121	—	—	—
7	0	—	—	.1734 ± .0122	—	—	—
8	0	—	—	-.2011 ± .0114	—	—	—

Table II - Gravity Field of Moon Determined from Lunar Orbiters (Tolson and Gapcynski, 1967; Lorell and Sjogren, 1968) and Luna-10 (Akim, 1966).

ORIENTATION OF PRINCIPAL AXES

A Cartesian set of coordinate axes fixed in the Moon are defined with the x and y axes in the equatorial plane, the x axis directed toward the mean direction to the Earth, and the z axis directed north along the axis of rotation. The second degree terms in the gravity field are related to the moments and products of inertia as follows:

$$I_{xx} = A \qquad\qquad\qquad I_{xy} = 2Ma^2 S_{2,2}$$

$$I_{yy} = B = A + 4Ma^2 C_{2,2} \qquad\qquad I_{xz} = Ma^2 C_{2,1}$$

$$I_{zz} = C = A + Ma^2(2C_{2,2} - C_{2,0}) \qquad I_{yz} = Ma^2 S_{2,1}$$

where A, B, and C are the Moon's principal moments of inertia, M is the Moon's mass, and a is the Moon's mean radius.

The second degree terms in the spherical harmonic representation of the distribution of surface features are related to the moments and products of inertia as follows:

$$I_{xx} = \frac{8\pi}{3}\, \sigma a^4 \left[C'_{0,0} + \frac{1}{10} C'_{2,0} - \frac{3}{5} C'_{2,2} \right] \qquad I_{xy} = \frac{8\pi}{5}\, \sigma a^4 S'_{2,2}$$

$$I_{yy} = \frac{8\pi}{3}\, \sigma a^4 \left[C'_{0,0} + \frac{1}{10} C'_{2,0} + \frac{3}{5} C'_{2,2} \right] \qquad I_{xz} = \frac{4\pi}{5}\, \sigma a^4 C'_{2,1}$$

$$I_{zz} = \frac{8\pi}{3}\, \sigma a^4 \left[C'_{0,0} - \frac{1}{5} C'_{2,0} \right] \qquad\qquad I_{yz} = \frac{4\pi}{5}\, \sigma a^4 S'_{2,1}$$

where σ is the load per unit area due to the higher elevation of continents and areas outside of circular basins.

The orientations of the principal axes of the continent distribution, the non-circular basin distributions, and the overall mass distribution expressed by the gravity potential coefficients indicated in Table III were determined by the above equations and the equations presented by Munk and MacDonald (1960b). The east longitudes of the axes of least and greatest moments of inertia in the equatorial plane

	Maximum moment of inertia		Intermediate moment of inertia		Minimum moment of inertia		Minimum moment of inertia in equatorial plane
	λ	ϕ	λ	ϕ	λ	ϕ	λ
Akim (1966)	16.0	176.0	273.1	91.0	3.0	86.1	2.8
Tolson and Gapcynski (1967)	353.5	176.1	274.6	89.3	4.7	86.2	4.8
Lorell and Sjogren (1968)	354.1	178.0	74.6	89.7	344.6	88.0	344.6
Distribution 1	17.3	109.2	278.6	113.3	322.8	31.0	295.5
Distribution 2	349.9	114.3	271.4	66.1	40.1	35.3	71.3
Distribution 3	2.8	112.5	275.6	85.6	12.4	32.9	89.3
Distribution 4	355.0	118.7	279.1	66.0	42.5	39.0	66.8

Table III - Orientations of principal axes determined from lunar gravity field and coefficients of spherical harmonic representations of distributions of surface features; λ is east longitude and ϕ is co-latitude in degrees.

related to the gravity field and the spherical harmonic representations of surface features were determined by $\tan 2\lambda = S_{2,2}/C_{2,2}$. The east longitudes corresponding to the axes of least moment of inertia on Table III were identified as representing the smaller value of the quantity: $- C_{2,2} \cos 2\lambda - 2S_{2,2} \sin 2\lambda$.

For stable orientation, the axis of greatest moment of inertia corresponds to the rotational axis and the axis of least moment lies along a line pointing to the Earth. The orientations of the principal axes, determined from the coefficients in the gravity field presented by Akim (1966) and Tolson and Gapcynski (1967), lie within 5 degrees of the stable orientation. For the coefficients presented by Lorell and Sjogren (1968), the axes lie within about 15 degrees of the stable orientation. The principal axes of the distributions of surface features are within 30 to 40 degrees of an orientation opposite to a stable orientation. That is, the axes of least moment of inertia lie within 40 degrees of the polar axis and the axes of greatest moment are within 30 degrees of the center of the disk. These orientations and the lower elevation of maria and circular basins verify the authors' earlier rejection of a homogeneous density distribution within the Moon. The effect of an excess load (σ) due to the greater elevation of continents and areas outside of circular basins is more than compensated for by excess mass beneath the maria and circular basins.

Significantly, Nash (1963) suggested that excess mass beneath the maria may be the explanation for the relative lack of maria on the Moon's farside. Nash hypothesized that the Moon was locked into synchronous rotation, with excess mass beneath the maria facing the Earth in a stable orientation. He also suggested that the maria resulted from the impact of objects denser than the Moon and that the apparent concentration of maria in the plane of the ecliptic may be explained by an asteroidal origin for the impacting objects. However, any process of maria formation which is random and leads to an excess of mass beneath the maria surface could result in the present distribution of maria with respect to the Moon's rotational axis and the direction to the Earth. The present distribution could be the result of wander of the Moon's axes to the position of stable equilibrium, regardless of the original cause of the density distribution.

MOMENT OF INERTIA

Tolson and Gapcynski (1967) have calculated the value of the quantity $q = 3C/2Ma^2 = 0.5895 \pm 0.05$ where C is the moment of inertia about the axis of rotation from their values of $C_{2,0}$ and $C_{2,2}$. According to Jeffreys (1962), for a chemically homogeneous moon with increase in density with depth due to internal pressure, $q = 0.596$. According to our calculations (Lamar and McGann, 1966a) for a linear increase in density from 3.2 gm/cm^3 below a thin crust to 3.76 gm/cm^3 at the center, $q = 0.592$, and for a more extreme differentiated model in which the density increases linearly from 2.8 gm/cm^3 at the surface to 5.0 gm/cm^3 at the center, $q = 0.568$. The problem of the overall moment of inertia has been considerably improved by the orbiter data and it is apparently no longer necessary to consider the possibility that the density of the Moon decreases with depth. However the data are still insufficient to decide whether the Moon is differentiated or chemically homogeneous.

INTERNAL STRUCTURE

In this section we consider internal density distributions which reconcile the coefficients of spherical harmonic expansions of the external gravity field with similar coefficients representing surface feature distributions. Arbitrary cutoffs are assumed between maria and continents and between circular basins and areas outside of circular basins. If such models reflected the true internal structure, the principal axes of the distributions of highlands (continents and areas outside of circular basins) would correspond to the Moon's principal axes. If excess mass beneath the maria or circular basins is greater than the load due to the higher elevation of continents and areas outside of basins, the Moon's axis of least moment of inertia will correspond to the axis of greatest moment for the distribution of highlands.

As noted in the previous section for the distributions considered, the principal axes of the surface features are no closer than 30 to 40 degrees to such an orientation. Because of this, and the preliminary nature of the coefficients of the gravity field, consideration of more elaborate models of internal structure is not now justified. Eventually it will be appropriate to consider models of internal structure in which the mass effects beneath individual basins is considered a function of the age and size of the feature. If the principal axes for such models correspond to the Moon's principal axes, it would be of interest to consider higher order terms in the gravity field and distribution of surface features.

In the present analysis, only the terms which reflect the oblateness $(C_{2,0})$, ellipticity of the equator $(C_{2,2})$, and north-south asymmetry $(C_{3,0})$ are considered. The calculations give an idea of the internal density variations required to reconcile the gravity field and the gross shape as reflected in the distribution of surface features. The following models of internal structure are considered:

(1) The Moon is assumed to be a rigid body not in isostatic equilibrium; the density variations are assumed to be the result of random inhomogeneities as suggested by Urey, Elsasser, and Rochester (1959).

(2) The Moon is assumed to be in isostatic equilibrium, and differences in elevation are assumed to be compensated for by variations in crustal thickness as suggested by O'Keefe and Cameron (1962). For this model the required density variations beneath the crust could be due to temperature differences related to thermal convection in the mantle, as proposed by Kopal (1962) and Runcorn (1962). From Jeffreys (1962, p. 182) the equation for the gravitational potential of a surface distribution over a sphere of radius R is

$$U = \frac{4\pi G \sigma R}{(2n + 1)} \sum_{n=2}^{\infty} \sum_{m=0}^{n} \left(\frac{R}{r}\right)^{n+1} P_{n,m}(\sin\phi)(C'_{n,m} \cos m\lambda + S'_{n,m} \sin m\lambda)$$

where G is the gravitational constant, σ is the surface area density over the sphere expressed in a spherical harmonic representation with coefficients $C'_{n,m}$ and $S'_{n,m}$, and r is the distance from the center of the sphere. From the equation for the lunar gravitational potential, we derive the following expression for the effects on individual coefficients of the gravity field:

$$C_{n,m} = \frac{4\pi\sigma R^{n+2} C'_{n,m}}{(2n + 1)Ma^n}$$

where M is the mass and a is the Moon's mean radius.

Assuming the Moon is homogeneous and substituting the expression for the mass of a homogeneous sphere, we derive

$$H = d_{n,m} a/3$$

where H is the height of a surface layer, a is the Moon's mean radius (1738 km), and $d_{n,m}$ is (2n + 1) times the ratio of the coefficient in the gravity field to a similar term in the distribution of the surface layer. The surface layer is assumed to have the same density as the Moon. Table IV shows the excess elevation (H) of maria and circular basins required to produce the gravity field determined by Tolson and Gapcynski (1967) from the distributions of surface features. However,

the available data indicate that maria and circular basins are low in elevation, thus these calculations indicate that the assumption of a homogeneous density distribution is false.

	Dist. 1	Dist. 2	Dist. 3	Dist. 4
$C_{2,0}$	4.1	7.1	13.5	23.1
$C_{2,2}$	2.6	2.5	4.5	10.2
$C_{3,0}$	1.5	8.5	4.2	6.0

Table IV - Excess elevation (km) of maria and circular basins required to produce coefficients in the lunar gravity field determined by Tolson and Gapcynski (1967) from distributions of surface features and a homogeneous Moon.

To estimate simple mass distributions which will produce the values of terms in the gravity field, it is necessary to derive general equations analogous to those presented in an earlier paper for the second order terms (Lamar and McGann, 1966a). In integral form the effect on individual coefficients in the gravity field is

$$C_{n,m} = \frac{3C'_{n,m}}{(2n + 1)\rho_a a^{n+3}} \int_0^{a+H} r^{n+2} \Delta\rho(r)dr$$

where H is the assumed excess elevation of continents and areas outside of circular basins, ρ_a is the Moon's average density (3.33 gms/cm^3), and $\Delta\rho(r)$ is the excess in density of material beneath maria and circular basins.

For a rigid, lumpy model, we assume a linear decrease in the density contrast with depth, or $\Delta\rho(r) = \Delta\rho(a)(r/a)$, where $\Delta\rho(a)$ is the density contrast at the surface; by integration

$$\Delta\rho(a) = \rho_a\left[\frac{d_{n,m}}{3} - \frac{H}{a}\right](n + 4)$$

It is assumed that the density of material near the surface is equal to the average density of the Moon.

For the extreme case of a rigid, lumpy model, we assume that maria and circular basins are directly underlain by a layer of nickel-iron. The thickness (t) of the layer is found to be

$$t = \frac{\rho_a}{(\rho_i - \rho_a)} \left[H - \frac{d_{n,m}a}{3} \right]$$

where ρ_i is the density of nickel iron (7.8 gm/cm^3).

If topographic irregularities are assumed to be compensated for by variations in thickness of crust floating on mantle, we assume zero thickness for crust beneath maria and circular basins, compensation at depth, D, and a density excess in mantle beneath circular basins and maria which decreases linearly with depth; by integration

$$\Delta\rho(a) = \frac{(n + 4)}{\left[1 - (n + 4) \frac{D}{a} \right]} \left[\frac{\rho_a d_{n,m}}{3} - (\rho_c - \rho_m) \frac{D}{a} - \rho_c \frac{H}{a} \right]$$

where ρ_c is the density of crustal material and ρ_m is the density of the mantle directly beneath maria and circular basins. We assume a condition of zero stress at the Moon's center; from Lamar and McGann (1966a)

$$\Delta\rho(a) = \frac{3}{\left(1 - 3 \frac{D}{a} \right)} \left[\rho_c \frac{H}{a} - (\rho_m - \rho_c) \frac{D}{a} \right]$$

We assume that the average density of crustal material is 2.8 gm/cm^3 and that the density contrast between the crust and mantle $(\rho_m - \rho_c)$ is 0.4 gm/cm^3. Accepted values for the Earth are $\rho_c = 2.84$ gm/cm^3 and $(\rho_m - \rho_c) = 0.43$ gm/cm^3 (Worzel and Shurbet, 1955). For the second order terms and the relations

$$C_{2,0} = -J_2^o = \frac{1}{Ma^2} \left[\frac{A + B}{2} - C \right], \qquad C'_{2,0} = a_2^o$$

$$C_{2,2} = -J_2^2 = \frac{1}{4Ma^2}(B - A), \quad C'_{2,2} = \frac{1}{2}a_2^2$$

$$\beta = \frac{C - A}{B} \simeq \frac{C - A}{C}, \quad \gamma = \frac{B - A}{C}, \quad C \simeq \frac{2}{5}Ma^2$$

where C, B, and A are the principal moments of inertia of the Moon, the above equations reduce to those presented in Lamar and McGann (1966a).

Table V lists values of parameters for the models of internal structure calculated by the above equations from the coefficients for distributions of surface features and Tolson and Gapcynski's (1967) determination of the gravity field.

		H (km)	Rigid Models		Isostatic Model	
			Density contrast, $\Delta\rho(a)$ (gm/cm^3)	Thickness of Nickel-Iron Layer, t (km)	Density Contrast, $\Delta\rho(a)$ (gm/cm^3)	Depth of Compensation, D (km)
Dist. 1	$c_{2,0}$	3.0	0.081	5.3	0.017	44.3
		5.0	0.10	6.8	0.018	58.5
	$c_{2,2}$	3.0	0.064	4.2	0.011	35.7
		5.0	0.087	5.7	0.011	49.9
	$c_{3,0}$	3.0	0.060	3.3	0.0064	29.8
		5.0	0.087	4.8	0.0066	43.9
Dist. 2	$c_{2,0}$	3.0	0.12	7.5	0.032	62.0
		5.0	0.14	9.0	0.033	76.5
	$c_{2,2}$	3.0	0.064	4.1	0.011	35.4
		5.0	0.087	5.6	0.011	49.6
	$c_{3,0}$	3.0	0.15	8.6	0.042	73.8
		5.0	0.18	10.1	0.044	88.6
Dist. 3	$c_{2,0}$	3.0	0.19	12.3	0.068	101.7
		5.0	0.21	13.8	0.071	116.9
	$c_{2,2}$	3.0	0.086	5.6	0.019	46.5
		5.0	0.11	7.0	0.020	60.7
	$c_{3,0}$	3.0	0.097	5.4	0.019	46.5
		5.0	0.12	6.9	0.020	60.8
Dist. 4	$c_{2,0}$	3.0	0.30	19.4	0.015	169.6
		5.0	0.32	20.9	0.016	187.4
	$c_{2,2}$	3.0	0.15	9.8	0.048	80.7
		5.0	0.17	11.3	0.050	95.5
	$c_{3,0}$	3.0	0.12	6.7	0.028	57.3
		5.0	0.15	8.2	0.029	71.8

Table V - Parameters for models of internal structure required to produce coefficients in the lunar gravity field determined by Tolson and Gapcynski (1967) from distributions of surface features. The quantity H represents assumed values of the excess elevation of continents and areas outside of circular basins.

CONCLUSIONS

An excess in density beneath the lowlands (maria and circular basins) is indicated by the relation between the $C_{2,0}$, $C_{2,2}$, and $C_{3,0}$ terms in the gravity field and the distribution of surface features (Table V). However, the density excesses corresponding to the three terms differ by no more than a factor of three for the three models of internal structure considered.

Distribution 4 most closely corresponds to the existence of "mascons" beneath the inner rims of the most youthful appearing circular basins, as revealed by the analysis of Muller and Sjogren (1968). If mascons are assumed to be masses of nickel-iron, they correspond to a layer averaging about 12 km thick, as determined from Table V, or more realistically, a layer several tens of kilometers in thickness with a 10-20% excess of nickel-iron.

The isostatic model fits the data equally well. The required depths of compensation for this model appear reasonable; they are not significantly greater than corresponding depths determined for the Earth. The required excess in the density of the mantle beneath circular basins and maria for the isostatic model could be explained by a lower temperature of a few hundred degrees centigrade.

Data from the Lunar Orbiter Program have greatly improved our knowledge of the Moon's gravity field and gross shape as reflected in the distribution of surface features. However, these data alone are insufficient to determine which model of internal structure is most probable, or the nature of the processes which have shaped the Moon's surface.

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Appendix
Computer Programs

This appendix contains listings, explanations and flow diagrams for all programs written for this project. The "Spherical Harmonics" program consists of a main routine and two subroutines. "MAIN" assumes the surface of the Moon to be divided into 10^0 squares for calculation of the spherical harmonic coefficients of the distribution of surface features. The subroutine "PRNT" produces a table of the distribution of the surface features as determined from maps of the Moon and is the input to the program. The subroutine "TENSR" generates the tensor for computing the moments of inertia about the principal axes, and their orientation. The "Moon Modeling" program computes the parameters for different models of internal structure.

Table 1--Symbols Used in "MAIN" routine of Spherical Harmonics Program

Fortran Nomenclature	Explanation
	Variables
RUN	Label for data identification
THTA	θ, colatitude
ELAM	λ, east longitude
F	$f(\theta,\lambda)$; continentality function
ELCϕ	$\Delta\lambda$
THCϕ	$\Delta\theta$
SITA	$\sin\theta$
CϕTA	$\cos\theta$
F1	$(m + r)!$
F2	$(n - r)!$
F3	$(n - m - r)!$
F4	$r!$
Q	$(\cos\theta - 1)^{(n-m-r)}$
R	$(\cos\theta + 1)^{r}$
HASU	$[(\cos\theta - 1)^{n-m-r}(\cos\theta + 1)^{r}]/[(m + r)!(n - r)!(n - m - r)!r!]$; one term of summation
FISU	$\displaystyle\sum_{i=0}^{n-m} \frac{(\cos\theta - 1)^{n-m-r}(\cos\theta + 1)^{r}}{(m + r)!(n - r)!(n - m - r)!r!}$
F5	$(n - m)!$
F6	$(n + m)!$
SM	$(\sin\theta)^{m}$
TTN	2^{n}

Table 1--continued

Fortran Nomenclature	Explanation
P	$$p_n^m(\cos\theta) = \frac{(n-m)!(n+m)!(\sin\theta)^m}{2^n}$$ $$\sum_{i=0}^{n-m} \frac{(\cos\theta-1)^{n-m-r}(\cos\theta+1)^r}{(m+r)!(n-r)!(n-m-r)!r!}$$
SIML	$\sin m\lambda$
CϕML	$\cos m\lambda$
THP	$[(p_n^m(\cos\theta)\sin\theta)_i + (p_n^m(\cos\theta)\sin\theta)_{i+1}]/2.$
APAL	$[(\cos m\lambda)_i + (\cos m\lambda)_{i+1}]/2.$
BPAL	$[(\sin m\lambda)_i + (\sin m\lambda)_{i+1}]/2.$
A	$f(\theta,\lambda)\frac{\left[(p_n^m\sin\theta)_i + (p_n^m\sin\theta)_{i+1}\right]}{2.}\frac{\left[(\cos m\lambda)_i + (\cos m\lambda)_{i+1}\right]\Delta\theta\Delta\lambda}{2.}$
B	$f(\theta,\lambda)\frac{\left[(p_n^m\sin\theta)_i + (p_n^m\sin\theta)_{i+1}\right]}{2.}\frac{\left[(\sin m\lambda)_i + (\sin m\lambda)_{i+1}\right]\Delta\theta\Delta\lambda}{2.}$
F7	$n!$
CϕNST	$(2n+1)n! / 2\pi(n+m)!$ or $(2n+1)/4\pi$
CSUM	summation of C terms
SSUM	summation of S terms
CNM = CIS	$c_{n,m}$ } coefficients of a surface spherical harmonic
SNM = SIS	$s_{n,m}$ } expansion of the gravity field
	Integers
ICC	number of continentality function distributions to be run
ICCK	run counter
IPRT	switch for requesting printout of table of continentality function
ITR	switch for requesting "TENSR" calculations

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Table 1--continued

Fortran Nomenclature	Explanation
NC	number of colatitudes considered
NL	number of east longitudes considered
NMN	number of n,m combinations considered
NMNI	counter for number of n,m combinations
LIM	upper limit for summation
IF1	$(m + r)$
IF2	$(n - r)$
IF3	$(n - m - r)$
IF4	r — last term of factorial
IF5	$(n - m)$
IF6	$(n + m)$
IF7	n

The entries IF1 through IF7 are bracketed together with the note "last term of factorial".

Table 2—Symbols Used in "TENSR" subroutine of Spherical Harmonics Program

Fortran Nomenclature	Explanation
Variables	
COO	$C_{0,0}$
C20	$C_{2,0}$
C21	$C_{2,1}$
C22	$C_{2,2}$
S21	$S_{2,1}$
S22	$S_{2,2}$
EM1	I_{xx}
EM2	I_{yy}
EM3	I_{zz} $\Big\}$ matrix elements
EM4	I_{xy}, I_{yx}
EM5	I_{xz}, I_{zx}
EM6	I_{yz}, I_{zy}
A	Coefficient of I^3
B	Coefficient of I^2
C	Coefficient of I
D	constant term of cubic
Q	
R	
TEST	
SI	Storage locations used in finding roots of cubic
P	
PART	
COEF	

Table 2--continued

Fortran Nomenclature	Explanation
Y1	
Y2	storage locations used in finding roots of cubic
Y3	
Z1	
Z2	roots of the cubic
Z3	
XPØLE	greatest moment of inertia, I_x
YPØLE	intermediate moment of inertia, I_y
ZPØLE	least moment of inertia, I_z
H1	$I_{xx} - I_x$
H2	$I_{yy} - I_x$ diagonal terms of orientation matrix
H3	$I_{zz} - I_x$
X3	ω_z/ω_x
X2	ω_y/ω_x direction numbers
X1	ω_x/ω_x
S1	$\left[(\omega_z/\omega_x)^2 + (\omega_y/\omega_x)^2 + (\omega_x/\omega_x)^2\right]^{\frac{1}{2}}$
CALP1	$\cos \alpha_1$
CBET1	$\cos \beta_1$ direction cosines
CGAM1	$\cos \gamma_1$
AML1	λ_1; east longitude (radians) of x-axis
TTHA1	$\tan \theta_1$
THA1	θ_1; colatitude (radians) of x-axis
AML11	λ_1; east longitude (degrees) of x-axis

<div align="center">Table 2--continued</div>

Fortran Nomenclature	Explanation
AML12	$\lambda_1 \pm 180^{\circ}$
THTA11	θ_1; colatitude (degrees) of x-axis
THTA12	$\theta_1 \pm 180^{\circ}$

<div align="center">(this pattern is repeated for the y and z axes)</div>

X	$C_{2,2}/S_{2,2}$
Y	$[(C_{2,2}/S_{2,2})^2 + 1]^{\frac{1}{2}}$
TP1	$\tan \phi_1 = C_{2,2}/S_{2,2} + [(C_{2,2}/S_{2,2})^2 + 1]^{\frac{1}{2}}$
PH1	ϕ_1; east longitude (radians)
SPH1	$\sin \phi_1$
CPH1	$\cos \phi_1$
P11	moment of inertia about ϕ_1
PHI1	ϕ_1; east longitude (degrees)
TP2	$\tan \phi_2 = C_{2,2}/S_{2,2} - [(C_{2,2}/S_{2,2})^2 + 1]^{\frac{1}{2}}$
PH2	ϕ_2; east longitude (radians)
SPH2	$\sin \phi_2$
CPH2	$\cos \phi_2$
P22	moment of inertia about ϕ_2
PHI2	ϕ_2; east longitude (degrees)
PHMIN	minimum value of moment of inertia

Table 2--continued

Fortran Nomenclature	Explanation
	Integers
ISN	number of sets of spherical harmonic inputs to be run
ISC	run counter
MIN	=1; P11 is a minimum =2; P22 is a minimum
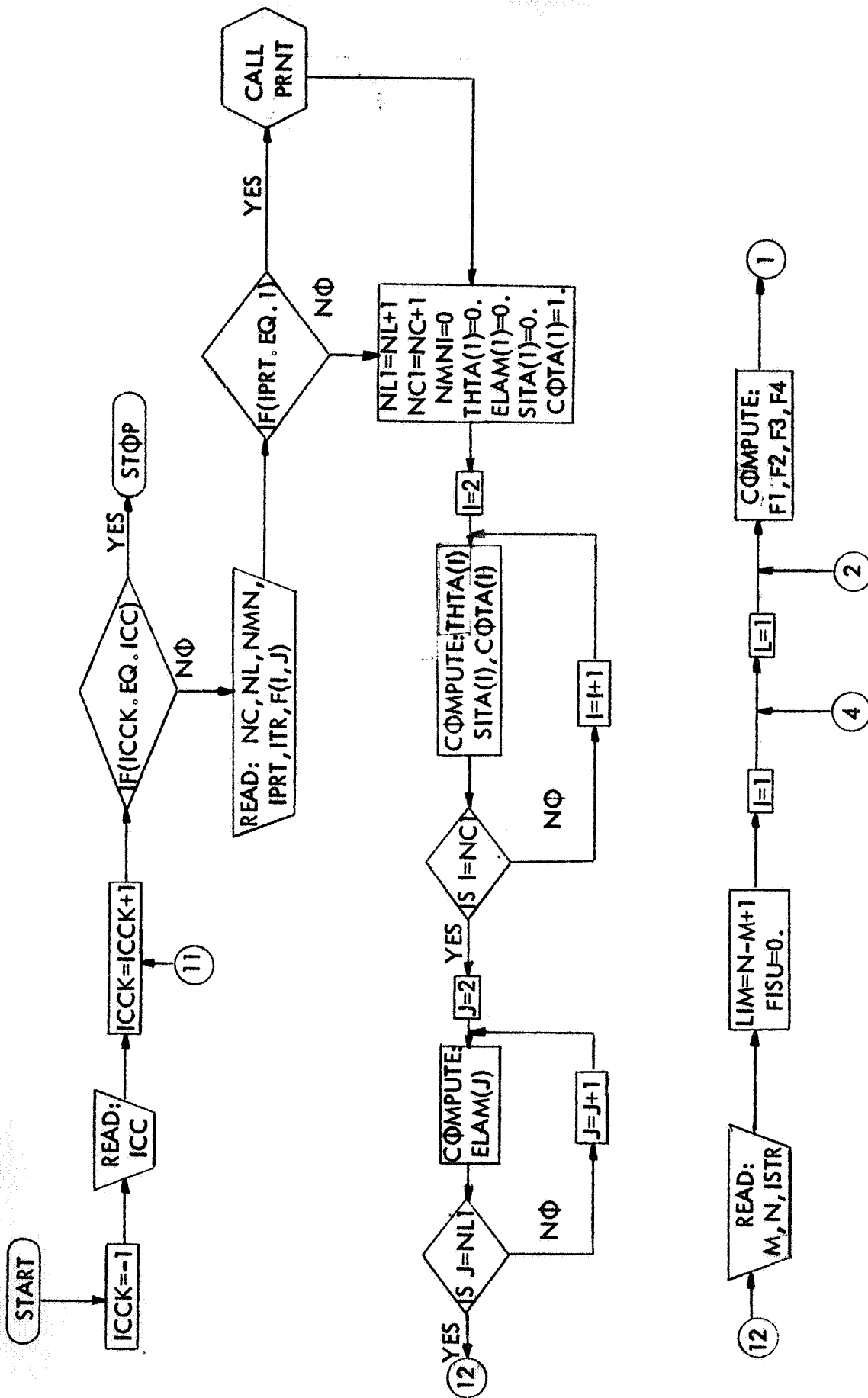
Table 3--Flow Diagram of "MAIN" routine of Spherical Harmonics Program

Table 3—Continued

③ → COMPUTE: F5, F6 → IF(M.EQ.0)

IF(M.EQ.0) — YES → SM(I)=1.

IF(M.EQ.0) — NO → IF(SITA(I).EQ.0)

IF(SITA(I).EQ.0) — YES → SM(I)=0.

IF(SITA(I).EQ.0) — NO → SM(I)=(SITA(I))^M

→ IF(N.EQ.0)

IF(N.EQ.0) — YES → TTN=1.

IF(N.EQ.0) — NO → TTN=2^N

→ COMPUTE: P(I), SET FISU=0.

→ IS I=NC1

IS I=NC1 — YES → ⑤

IS I=NC1 — NO → I=I+1 → ④

⑤ → J=1 → COMPUTE: SIML(J), COML(J) → IS J=NL1

IS J=NL1 — YES → I=1 → COMPUTE: THP(I) → IS I=NC

IS J=NL1 — NO → J=J+1

IS I=NC — YES → ⑥

IS I=NC — NO → I=I+1

Table 3--Continued

Table 3—Continued

Table 3—Continued

Table 3--Continued

```
                    ┌─────────┐
                    │  ENTRY  │
                    └────┬────┘
                         │
                         ▼
              ┌──────────────────────────┐
              │ C00=CIS(1)   C22=CIS(4)  │
              │ C20=CIS(2)   S21=SIS(3)  │
              │ C21=CIS(3)   S22=SIS(4)  │
              └────────────┬─────────────┘
                           │
```

WRITE: C00, C20, C21, C22, S21, S22

COMPUTE: EM1, EM2, EM3, EM4, EM5, EM6

WRITE: EM1, EM2, EM3, EM4, EM5, EM6 → (1)

(1)

COMPUTE: A, B, C, D, Q, R, TEST

COMPUTE: P, PART ← − IF(TEST) + → WRITE: ERROR MESSAGE

0

RETURN

COMPUTE: S1, Z1, Z2, Z3, XPOLE, YPOLE, ZPOLE

CØEF = −2×SQRT(−Q) ← − IF(R) + → CØEF = 2×SQRT(−Q)

0

WRITE: ERROR MESSAGE

RETURN

COMPUTE: Y1, Y2, Y3, Z1, Z2, Z3, AMIN, BMAX

(2)

(3)

Table 4—Flow Diagram of "TENSR" subroutine of Spherical Harmonics Program

②

IF(AMIN.EQ.Z1.AND.BMAX.EQ.Z2) ——YES—→ YPΦLE=Z3

NΦ

IF(AMIN.EQ.Z1.AND.BMAX.EQ.Z3) ——YES—→ YPΦLE=Z2

NΦ

IF(AMIN.EQ.Z2.AND.BMAX.EQ.Z1) ——YES—→ YPΦLE=Z3

NΦ

IF(AMIN.EQ.Z2.AND.BMAX.EQ.Z3) ——YES—→ YPΦLE=Z1

NΦ

IF(AMIN.EQ.Z3.AND.BMAX.EQ.Z1) ——YES—→ YPΦLE=Z2

NΦ

IF(AMIN.EQ.Z3.AND.BMAX.EQ.Z2) ——YES—→ YPΦLE=Z1

NΦ

XPΦLE=AMIN
ZPΦLE=BMAX

③

Table 4--Continued

(3)

COMPUTE:
H1,H2,H3,X3,X2,X1,
H4,H5,H6,X6,X5,X4,
H7,H8,H9,X9,X8,X7,
S1,CALP1,CBET1,CGAM1,
S2,CALP2,CBET2,CGAM2,
S3,CALP3,CBET3,CGAM3,
AML1,AML2,AML3,
TTHA1,THA1

IF(TTHA1) — → THA1=π-THA1

0,+

COMPUTE:
TTHA2,THA2

IF(TTHA2) — → THA2=π-THA2

0,+

COMPUTE:
TTHA3,THA3

IF(TTHA3) — → THA3=π-THA3

0,+

(4)

(4)

COMPUTE:
AML11,AML12

IF(AML11) — → AML11=2π+AML11
AML12=AML11-π

0,+

COMPUTE:
AML21,AML22

IF(AML21) — → AML21=2π+AML21
AML22=AML21-π

0,+

COMPUTE:
AML31,AML32

IF(AML31) — → AML31=2π+AML31
AML32=AML31-π

0,+

(5)

Table 4—Continued

⑤

```
COMPUTE:
THTA11, THTA12,
THTA21, THTA22,
THTA31, THTA32,
X, Y, TP1, PH1,
SPH1, CPH1, P11,
PHI1, TP2, PH2,
SPH2, CPH2, P22,
PHI2, PHMIN
```

IF(PHMIN.EQ.P11) —— YES → MIN=1

NØ

IF(PHMIN.EQ.P22) —— YES → MIN=2

NØ

```
WRITE:
XPØLE, AML11, THTA11, AML12, THTA12,
YPØLE, AML21, THTA21, AML22, THTA22,
ZPØLE, AML31, THTA31, AML32, THTA32
```

GØ TØ(1015, 1016), MIN —— =1 →
```
WRITE:
P22, PHI2,
P11, PHI1
```

=2

```
WRITE:
P11, PHI1,
P22, PHI2
```
→ RETURN

Table 4—Continued

```
C     SPHERICAL HARMONICS   10 DEGREE SQUARES
      DIMENSION THTA(19),SITA(19),COTA(19),ELAM( 37),B(18, 36),SM(19),
     A19),COML( 37),SIML( 37),THP(18),APAL( 36),BPAL( 36),A(18, 36)
      COMMON F(18,36),RUN(18),CIS(4),SIS(4)
C NC=NO OF VALUES OF COLAT CONSIDERED, NL=NO OF EAST LONG
      ICCK=-1
      READ(5,7000) ICC
 7000  FORMAT(I4)
 3050 ICCK=ICCK+1
      IF(ICCK.EQ.ICC) CALL EXIT
      READ(5,2001) (RUN(ID),ID=1,18)
 2001 FORMAT(18A4)
      READ(5,2003) NC,NL,NMN,IPRT,ITR
 2003 FORMAT(5I4)
      READ(5,2033) ((F(I,J),J=1,NL),I=1,NC)
 2033 FORMAT(18F4.2)
      IF(IPRT.EQ.1) CALL PRNT
      WRITE(6,2039)
 2039 FORMAT(1H1,11X,35HCOEFFICIENTS OF SPHERICAL HARMONICS)
      WRITE(6,2040)
 2040 FORMAT(1H0,11X,1HN,7X,1HM,9X,3HCNM,11X,3HSNM)
      NL1=NL+1
      NC1=NC+1
      NMNI=0
      ELCO=6.28318/FLOAT(NL)
      THCO=3.14159/FLOAT(NC)
      THTA(1)=0.
      ELAM(1)=0.
      SITA(1)=0.
      COTA(1)=1.
C FORM SERIES OF THETAS, LAMBDAS,  SINES AND COSINES FOR P CALC.
      DO 100 I=2,NC1
      THTA(I)=THTA(I-1)+THCO
      SITA(I)=SIN(THTA(I))
  100 COTA(I)=COS(THTA(I))
      DO 200 J=2,NL1
  200 ELAM(J)=ELAM(J-1)+ELCO
C FORM SUMMATION PART OF P, RELATED TO ACTUAL THETAS, WHEN R=0, SUBS=1
C GET VALUE OF M AND N, FORM ALL FACTORIALS, THEN SUM, THEN P
 3000 READ(5,2028) N,M,ISTR
 2028 FORMAT(3I4)
      LIM=N-M+1
      FISU=0.
      DO 400 I=1,NC1
      DO 500 L=1,LIM
      IF1=M+(L-1)
      IF(IF1.EQ.0) GO TO 1
      IFX1=1
      DO 2 II=1,IF1
    2 IFX1=IFX1*II
      F1=IFX1
  104 IF2=N-(L-1)
      IF(IF2.EQ.0) GO TO 3
      IFX2=1
      DO 4 II=1,IF2
    4 IFX2=IFX2*II
```

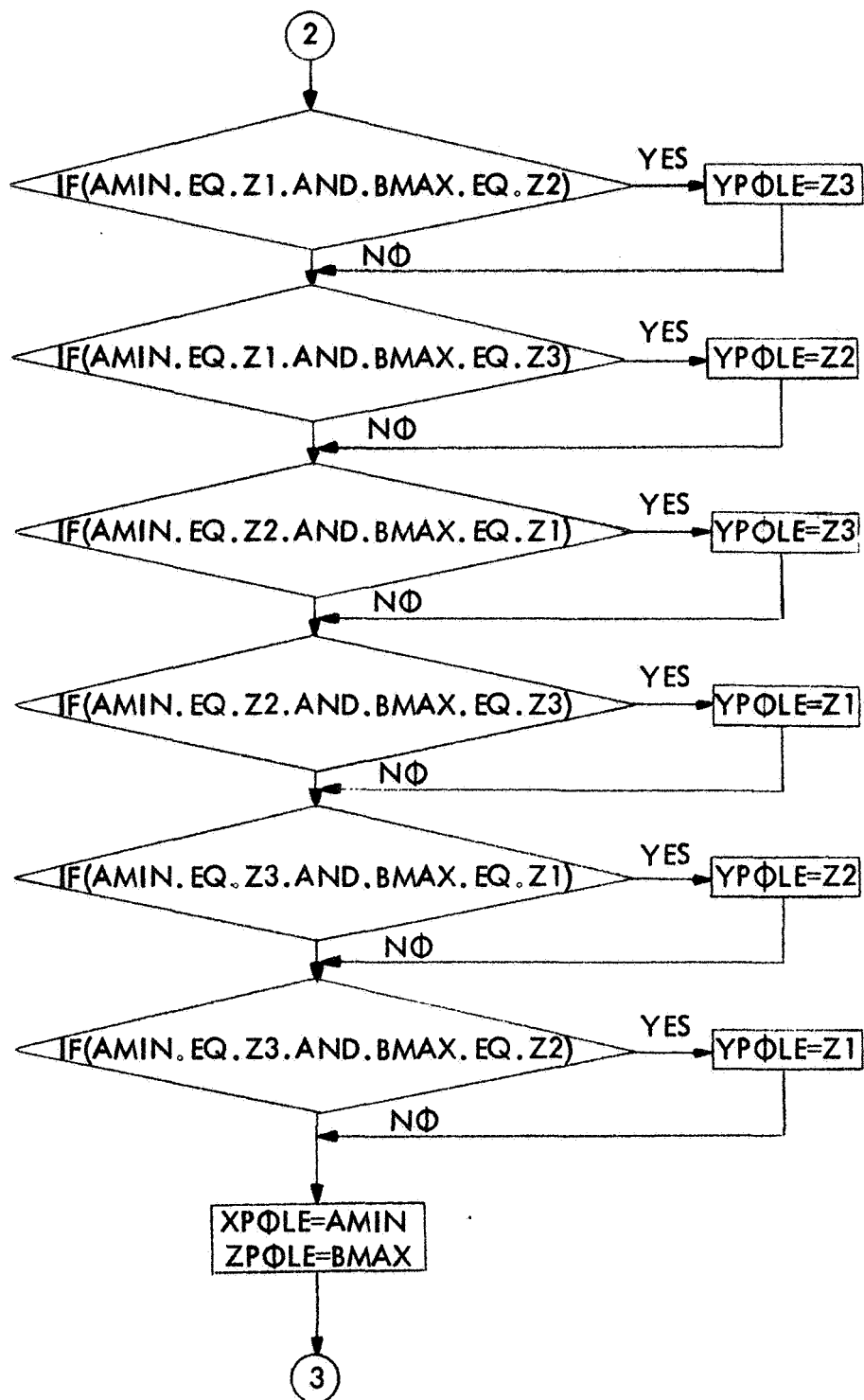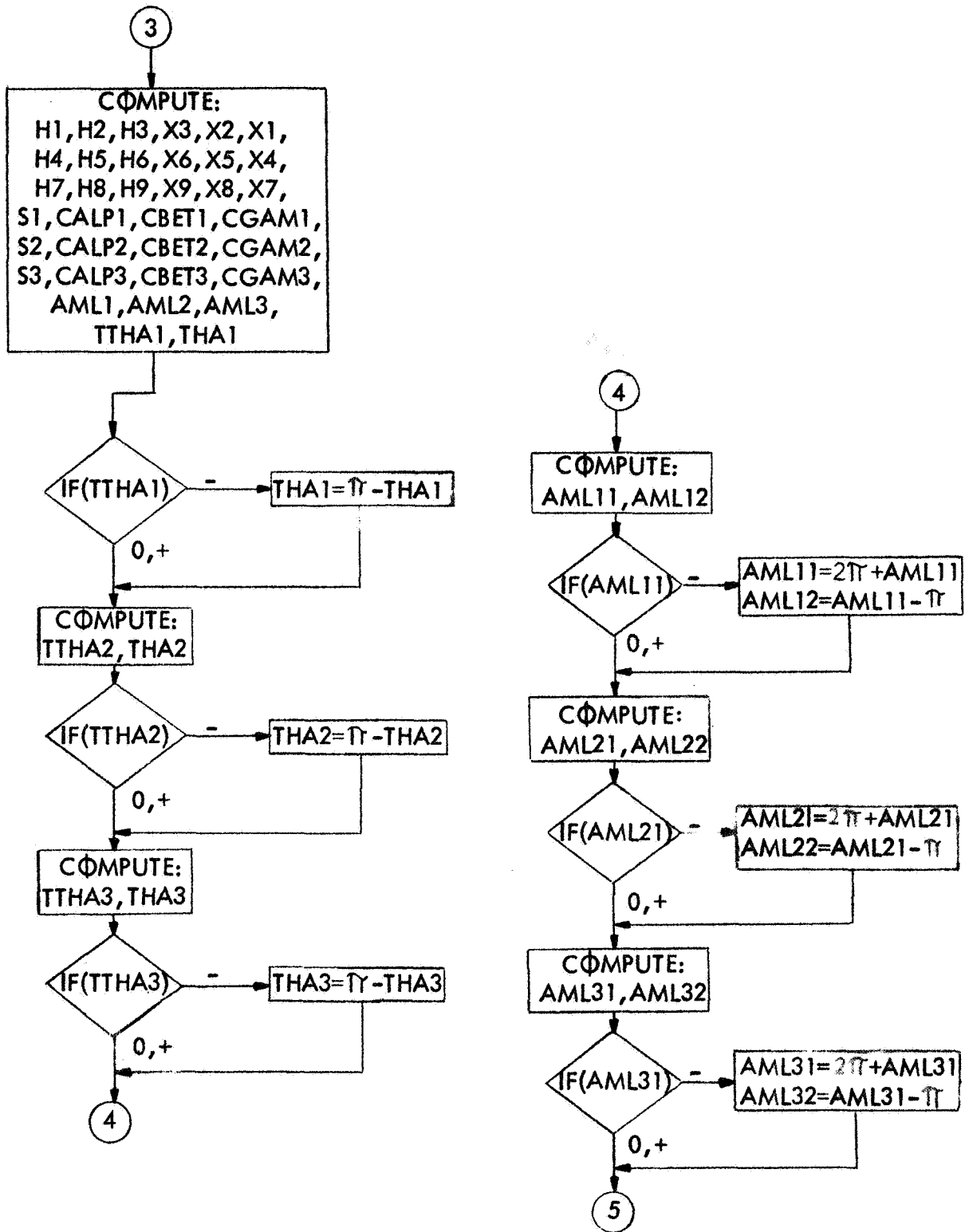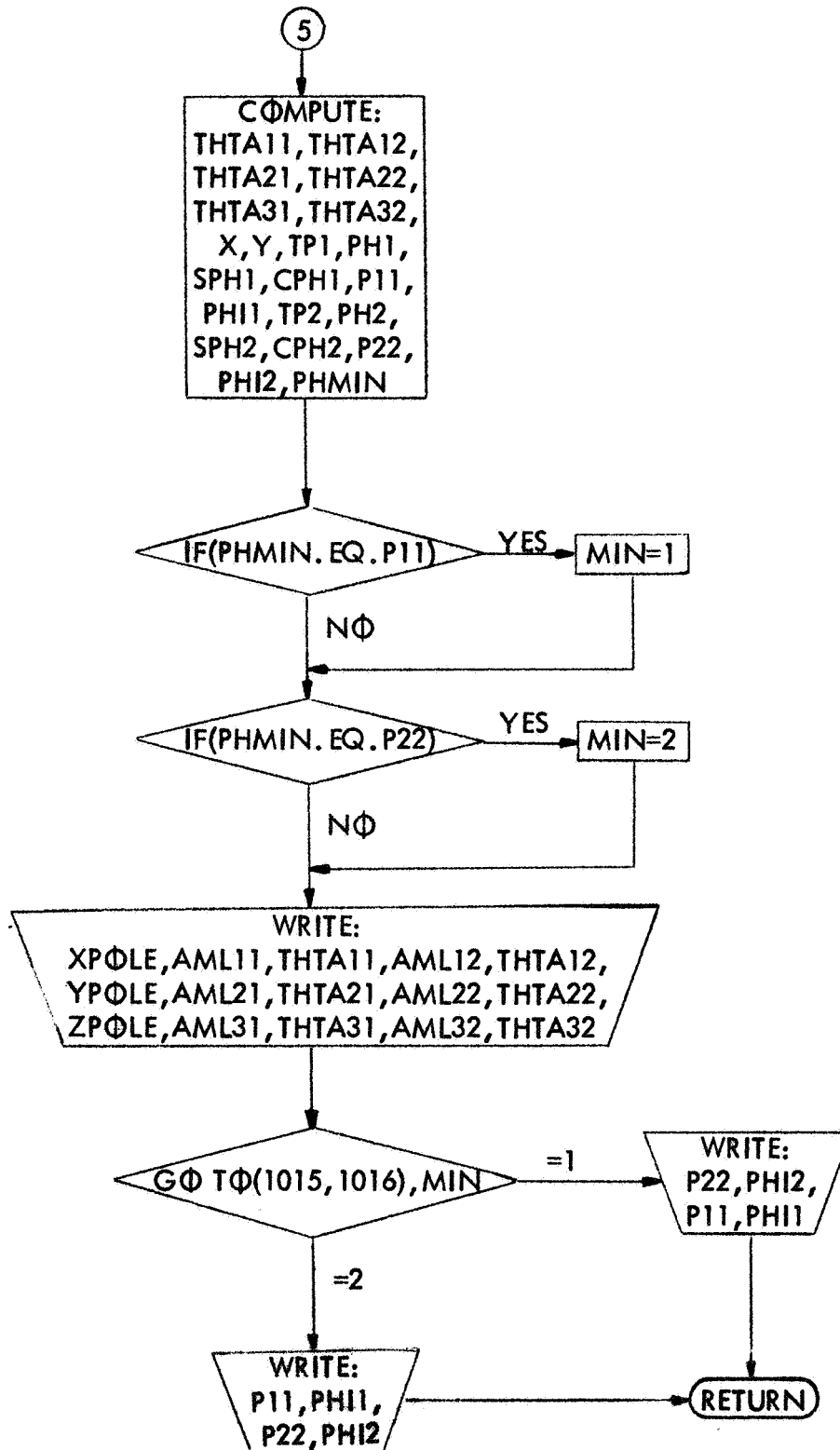Table 5--Listing of "MAIN" routine of Spherical Harmonics Program

```
      F2=IFX2
  106 IF3=N-M-(L-1)
      IF(IF3.EQ.0) GO TO 5
      IFX3=1
      DO 6 II=1,IF3
    6 IFX3=IFX3*II
      F3=IFX3
  108 IF4=L-1
      IF(IF4.EQ.0) GO TO 7
      IFX4=1
      DO 8 II=1,IF4
    8 IFX4=IFX4*II
      F4=IFX4
      GO TO 9
    1 F1=1.
      GO TO 104
    3 F2=1.
      GO TO 106
    5 F3=1.
      GO TO 108
    7 F4=1.
    9 IF((N-M-(L-1)).EQ.0) GO TO 10
      IF((COTA(I)-1.).EQ.0.) GO TO 111
      Q=(COTA(I)-1.)**(N-M-(L-1))
      GO TO 11
  111 Q=0.
      GO TO 11
   10 Q=1.
   11 IF((L-1).EQ.0) GO TO 12
      IF((COTA(I)+1.).EQ.0.) GO TO 112
      R=(COTA(I)+1.)**(L-1)
      GO TO 13
  112 R=0.
      GO TO 13
   12 R=1.
   13 HASU=(Q*R)/(F1*F2*F3*F4)
      FISU=FISU+HASU
  500 CONTINUE
C SUMMATION PART OF P NOW COMPLETE
      IF5=N-M
      IF(IF5.EQ.0) GO TO 14
      IFX5=1
      DO 15 II=1,IF5
   15 IFX5=IFX5*II
      F5=IFX5
  110 IF6=N+M
      IF(IF6.EQ.0) GO TO 16
      IFX6=1
      DO 17 II=1,IF6
   17 IFX6=IFX6*II
      F6=IFX6
      GO TO 401
   14 F5=1.
      GO TO 110
   16 F6=1.
  401 IF(M.EQ.0) GO TO 402
```

Table 5--Continued

```
      IF(SITA(I).EQ.0.) GO TO 403
      SM(I)=SITA(I)**M
      GO TO 404
  402 SM(I)=1.
      GO TO 404
  403 SM(I)=0.
  404 IF(N.EQ.0) GO TO 405
      TTN=2**N
      GO TO 406
  405 TTN=1.
  406 P(I)=F5*F6*SM(I)*FISU/TTN
      FISU=0.
  400 CONTINUE
C ALL PS ARE CAL. AND STORED RELATIVE TO THETAS AT THIS POINT
      EM=M
      DO 600 J=1,NL1
      SIML(J)=SIN(EM*ELAM(J))
  600 COML(J)=COS(EM*ELAM(J))
      DO 700 I=1,NC
  700 THP(I)=(P(I)*SITA(I)+P(I+1)*SITA(I+1))/2.
      IF(M.EQ.0) GO TO 801
      DO 800 J=1,NL
      APAL(J)=(COML(J)+COML(J+1))/2.
  800 BPAL(J)=(SIML(J)+SIML(J+1))/2.
      GO TO 901
  801 DO 802 J=1,NL
      APAL(J)=1.
  802 BPAL(J)=0.
  901 DO 900 I=1,NC
      DO 900 J=1,NL
      A(I,J)=F(I,J)*THP(I)*APAL(J)*THCO*ELCO
  900 B(I,J)=F(I,J)*THP(I)*BPAL(J)*THCO*ELCO
      IF(M.EQ.0) GO TO 21
      IF7=N
      IF(IF7.EQ.0) GO TO 18
      IFX7=1
      DO 19 II=1,IF7
   19 IFX7=IFX7*II
      F7=IFX7
      GO TO 20
   18 F7=1.
   20 CONST=(FLOAT(2*N+1)*F7)/(6.28319*F6)
      GO TO 22
   21 CONST=(FLOAT(2*N+1))/12.56636
   22 CSUM=0.
      SSUM=0.
      DO 1000 I=1,NC
      DO 1000 J=1,NL
      CSUM=CSUM+A(I,J)
 1000 SSUM=SSUM+B(I,J)
      CMN=CSUM*CONST
      SMN=SSUM*CONST
      WRITE(6,2041) N,M,CMN,SMN
 2041 FORMAT(1H0,10X,I2,6X,I2,4X,1PE12.5,3X,1PE12.5)
      IF(ISTR.EQ.0)GO TO 3020
      CIS(ISTR)=CMN
```

Table 5--Continued

```
      SIS(ISTR)=SMN
3020  NMNI=NMNI+1
      IF(NMNI.EQ.NMN.AND.ITR .EQ.1) CALL TENSR
      IF(NMNI.EQ.NMN) GO TO 3050
      GO TO 3000
      END
```

Table 5--Continued

```
      SUBROUTINE PRNT
C     SPHERICAL HARMONICS  10 DEGREE SQUARES
      COMMON F(18,36),RUN(18),CIS(4),SIS(4)
2005 FORMAT(1H ,10X,1H0)
2006 FORMAT(1H ,9X,2H10)
2007 FORMAT(1H ,9X,2H20)
2008 FORMAT(1H ,9X,2H30)
2009 FORMAT(1H ,9X,2H40)
2025 FORMAT(1H ,5X,1HC,7X,9(F4.2,2X))
2010 FORMAT(1H ,5X,1H0,3X,2H50)
2026 FORMAT(1H ,5X,1HL,7X,9(F4.2,2X))
2011 FORMAT(1H ,5X,1HA,3X,2H60)
2027 FORMAT(1H ,5X,1HT,7X,9(F4.2,2X))
2012 FORMAT(1H ,5X,1HI,3X,2H70)
2013 FORMAT(1H ,5X,1HU,3X,2H80)
2029 FORMAT(1H ,5X,1HD,7X,9(F4.2,2X))
2014 FORMAT(1H ,5X,1HE,3X,2H90)
2030 FORMAT(1H ,13X,9(F4.2,2X))
2015 FORMAT(1H ,5X,1H-,2X,3H100)
2016 FORMAT(1H ,5X,1HT,2X,3H110)
2031 FORMAT(1H ,5X,1HH,7X,9(F4.2,2X))
2017 FORMAT(1H ,5X,1HE,2X,3H120)
2018 FORMAT(1H ,5X,1HA,2X,3H130)
2019 FORMAT(1H ,8X,3H140)
2020 FORMAT(1H ,8X,3H150)
2021 FORMAT(1H ,8X,3H160)
2022 FORMAT(1H ,8X,3H170)
2023 FORMAT(1H ,8X,3H180)
      WRITE(6,2002)(RUN(ID),ID=1,17)
2002 FORMAT(1H1,17A4)
      WRITE(6,2034)
2034 FORMAT(1H0,28X,22HEAST LONGITUDE - LAMDA)
      WRITE(6,2035)
2035 FORMAT(1H0,12X,1H0,4X,2H10,4X,2H20,4X,2H30,4X,2H40,4X,2H50,4X,2H
     A,4X,2H70,4X,2H80,4X,2H90)
      WRITE(6,2005)
      WRITE(6,2030) (F(1,I),I=1,9)
      WRITE(6,2006)
      WRITE(6,2030) (F(2,I),I=1,9)
      WRITE(6,2007)
      WRITE(6,2030) (F(3,I),I=1,9)
      WRITE(6,2008)
      WRITE(6,2030) (F(4,I),I=1,9)
      WRITE(6,2009)
      WRITE(6,2025) (F(5,I),I=1,9)
      WRITE(6,2010)
      WRITE(6,2026) (F(6,I),I=1,9)
      WRITE(6,2011)
      WRITE(6,2027) (F(7,I),I=1,9)
      WRITE(6,2012)
      WRITE(6,2027) (F(8,I),I=1,9)
      WRITE(6,2013)
      WRITE(6,2029) (F(9,I),I=1,9)
      WRITE(6,2014)
      WRITE(6,2030) (F(10,I),I=1,9)
      WRITE(6,2015)
```

Table 6--Listing of "PRNT" subroutine of Spherical Harmonics Program

```
      WRITE(6,2030) (F(11,I),I=1,9)
      WRITE(6,2016)
      WRITE(6,2031) (F(12,I),I=1,9)
      WRITE(6,2017)
      WRITE(6,2027) (F(13,I),I=1,9)
      WRITE(6,2018)
      WRITE(6,2030) (F(14,I),I=1,9)
      WRITE(6,2019)
      WRITE(6,2030) (F(15,I),I=1,9)
      WRITE(6,2020)
      WRITE(6,2030) (F(16,I),I=1,9)
      WRITE(6,2021)
      WRITE(6,2030) (F(17,I),I=1,9)
      WRITE(6,2022)
      WRITE(6,2030) (F(18,I),I=1,9)
      WRITE(6,2023)
      WRITE(6,2002)(RUN(ID),ID=1,17)
      WRITE(6,2034)
      WRITE(6,2036)
 2036 FORMAT(1H0,11X,2H90,3X,3H100,3X,3H110,3X,3H120,3X,3H130,3X,3H140,
     1X,3H150,3X,3H160,3X,3H170,3X,3H180)
      WRITE(6,2005)
      WRITE(6,2030) (F(1,I),I=10,18)
      WRITE(6,2006)
      WRITE(6,2030) (F(2,I),I=10,18)
      WRITE(6,2007)
      WRITE(6,2030) (F(3,I),I=10,18)
      WRITE(6,2008)
      WRITE(6,2030) (F(4,I),I=10,18)
      WRITE(6,2009)
      WRITE(6,2025) (F(5,I),I=10,18)
      WRITE(6,2010)
      WRITE(6,2026) (F(6,I),I=10,18)
      WRITE(6,2011)
      WRITE(6,2027) (F(7,I),I=10,18)
      WRITE(6,2012)
      WRITE(6,2027) (F(8,I),I=10,18)
      WRITE(6,2013)
      WRITE(6,2029) (F(9,I),I=10,18)
      WRITE(6,2014)
      WRITE(6,2030) (F(10,I),I=10,18)
      WRITE(6,2015)
      WRITE(6,2030) (F(11,I),I=10,18)
      WRITE(6,2016)
      WRITE(6,2031) (F(12,I),I=10,18)
      WRITE(6,2017)
      WRITE(6,2027) (F(13,I),I=10,18)
      WRITE(6,2018)
      WRITE(6,2030) (F(14,I),I=10,18)
      WRITE(6,2019)
      WRITE(6,2030) (F(15,I),I=10,18)
      WRITE(6,2020)
      WRITE(6,2030) (F(16,I),I=10,18)
      WRITE(6,2021)
      WRITE(6,2030) (F(17,I),I=10,18)
      WRITE(6,2022)
```

Table 6---Continued

```
      WRITE(6,2030) (F(18,I),I=10,18)
      WRITE(6,2023)
      WRITE(6,2002)(RUN(ID),ID=1,17)
      WRITE(6,2034)
      WRITE(6,2037)
2037 FORMAT(1H0,10X,3H180,3X,3H190,3X,3H200,3X,3H210,3X,3H220,3X,3H230
    13X,3H240,3X,3H250,3X,3H260,3X,3H270)
      WRITE(6,2005)
      WRITE(6,2030) (F(1,I),I=19,27)
      WRITE(6,2006)
      WRITE(6,2030) (F(2,I),I=19,27)
      WRITE(6,2007)
      WRITE(6,2030) (F(3,I),I=19,27)
      WRITE(6,2008)
      WRITE(6,2030) (F(4,I),I=19,27)
      WRITE(6,2009)
      WRITE(6,2025) (F(5,I),I=19,27)
      WRITE(6,2010)
      WRITE(6,2026) (F(6,I),I=19,27)
      WRITE(6,2011)
      WRITE(6,2027) (F(7,I),I=19,27)
      WRITE(6,2012)
      WRITE(6,2027) (F(8,I),I=19,27)
      WRITE(6,2013)
      WRITE(6,2029) (F(9,I),I=19,27)
      WRITE(6,2014)
      WRITE(6,2030) (F(10,I),I=19,27)
      WRITE(6,2015)
      WRITE(6,2030) (F(11,I),I=19,27)
      WRITE(6,2016)
      WRITE(6,2031) (F(12,I),I=19,27)
      WRITE(6,2017)
      WRITE(6,2027) (F(13,I),I=19,27)
      WRITE(6,2018)
      WRITE(6,2030) (F(14,I),I=19,27)
      WRITE(6,2019)
      WRITE(6,2030) (F(15,I),I=19,27)
      WRITE(6,2020)
      WRITE(6,2030) (F(16,I),I=19,27)
      WRITE(6,2021)
      WRITE(6,2030) (F(17,I),I=19,27)
      WRITE(6,2022)
      WRITE(6,2030) (F(18,I),I=19,27)
      WRITE(6,2023)
      WRITE(6,2002)(RUN(ID),ID=1,17)
      WRITE(6,2034)
      WRITE(6,2038)
2038 FORMAT(1H0,10X,3H270,3X,3H280,3X,3H290,3X,3H300,3X,3H310,3X,3H320
    13X,3H330,3X,3H340,3X,3H350,3X,3H360)
      WRITE(6,2005)
      WRITE(6,2030) (F(1,I),I=28,36)
      WRITE(6,2006)
      WRITE(6,2030) (F(2,I),I=28,36)
      WRITE(6,2007)
      WRITE(6,2030) (F(3,I),I=28,36)
      WRITE(6,2008)
```

Table 6--Continued

```
      WRITE(6,2030) (F(4,I),I=28,36)
      WRITE(6,2009)
      WRITE(6,2025) (F(5,I),I=28,36)
      WRITE(6,2010)
      WRITE(6,2026) (F(6,I),I=28,36)
      WRITE(6,2011)
      WRITE(6,2027) (F(7,I),I=28,36)
      WRITE(6,2012)
      WRITE(6,2027) (F(8,I),I=28,36)
      WRITE(6,2013)
      WRITE(6,2029) (F(9,I),I=28,36)
      WRITE(6,2014)
      WRITE(6,2030) (F(10,I),I=28,36)
      WRITE(6,2015)
      WRITE(6,2030) (F(11,I),I=28,36)
      WRITE(6,2016)
      WRITE(6,2031) (F(12,I),I=28,36)
      WRITE(6,2017)
      WRITE(6,2027) (F(13,I),I=28,36)
      WRITE(6,2018)
      WRITE(6,2030) (F(14,I),I=28,36)
      WRITE(6,2019)
      WRITE(6,2030) (F(15,I),I=28,36)
      WRITE(6,2020)
      WRITE(6,2030) (F(16,I),I=28,36)
      WRITE(6,2021)
      WRITE(6,2030) (F(17,I),I=28,36)
      WRITE(6,2022)
      WRITE(6,2030) (F(18,I),I=28,36)
      WRITE(6,2023)
      RETURN
      END
```

Table 6--Continued

```
          SUBROUTINE TENSR
C      TENSOR OF INERTIA
          COMMON F(18,36),RUN(18),CIS(4),SIS(4)
          C00=CIS(1)
          C20=CIS(2)
          C21=CIS(3)
          C22=CIS(4)
          S21=SIS(3)
          S22=SIS(4)
    2     FORMAT(1H ,76HTEST OF Q**3+P**2 YIELDS A POSITIVE VALUE - ONE REAL
         1 AND TWO IMAGINARY ROOTS)
    3     FORMAT(1H ,58HR IS NEGATIVE, CALCULATION STOPPED, RETURN TO MAIN P
         1ROGRAM)
          PRINT 1000
 1000     FORMAT(1H1,31HSPHERICAL HARMONIC COEFFICIENTS)
          PRINT 1001
 1001     FORMAT(1H0,7X,3HC00,7X,3HC20,7X,3HC21,7X,3HC22,7X,3HS21,7X,3HS22)
 1002     FORMAT(1H ,2X,6(3X,F7.4),/)
          WRITE(6,1002) C00,C20,C21,C22,S21,S22
          EM1=C20
          EM2=4.*C22+C20
          EM3=2.*C22
          EM4=2.*S22
          EM5=C21
          EM6=S21
  101     PRINT 90
   90     FORMAT(1H0,15HMATRIX ELEMENTS)
          PRINT 91
   91     FORMAT(1H0,7X,3HM11,7X,3HM22,7X,3HM33,5X,7HM12=M21,3X,7HM13=M31,3X
         1,7HM23=M32)
          WRITE(6,1002) EM1,EM2,EM3,EM4,EM5,EM6
          PRINT 107
  107     FORMAT(1H0,62HPRINCIPAL MOMENTS   EAST LONG   COLATITUDE EAST LONG
         1COLATITUDE)
          PRINT 108
  108     FORMAT(1H ,60H   OF INERTIA       (DEG)      (DEG)      (DEG)
         1 (DEG) )
          A=1.
          B=-(EM1+EM2+EM3)/3.
          C=-(-EM1*EM2-EM1*EM3-EM2*EM3+EM4**2+EM5**2+EM6**2)/3.
          D=-(EM1*EM2*EM3+2.*EM4*EM5*EM6-EM1*EM6**2-EM2*EM5**2-EM3*EM4**2)
          Q=A*C-P**2
          R=.5*(3.*A*B*C-A**2*D)-B**3
          TEST=Q**3+R**2
          IF(TEST) 12,13,14
   14     PRINT 2
          GO TO 32
   13     SI=R**(1./3.)
          Z1=(2.*SI-D)/A
          Z2=(-SI-B)/A
          Z3=Z2
          XPOLE=AMIN1(Z1,Z2)
          ZPOLE=AMAX1(Z1,Z2)
          YPOLE=ZPOLE
          GO TO 199
   12     P=ABS(R)/SQRT(-Q**3)
```

Table 7—Listing of "TENSR" subroutine of Spherical Harmonics Program

```
      PART=ATAN(SQRT(1.-P**2)/P)/3.
      IF(R) 15,16,17
  16 PRINT 3
      GO TO 32
  15 COFF=-2.*SQRT(-Q)
      GO TO 102
  17 COEF=2.*SQRT(-Q)
 102 Y1=COEF*COS(PART)
      Y2=COEF*COS(PART+2.094395)
      Y3=COEF*COS(PART+4.18879)
      Z1=(Y1-B)/A
      Z2=(Y2-B)/A
      Z3=(Y3-B)/A
  19  AMIN=AMIN1(Z1,Z2,Z3)
      BMAX=AMAX1(Z1,Z2,Z3)
      IF(AMIN.EQ.Z1.AND.BMAX.EQ.Z2) YPOLE=Z3
      IF(AMIN.EQ.Z1.AND.BMAX.EQ.Z3) YPOLE=Z2
      IF(AMIN.EQ.Z2.AND.BMAX.EQ.Z1) YPOLE=Z3
      IF(AMIN.EQ.Z2.AND.BMAX.EQ.Z3) YPOLE=Z1
      IF(AMIN.EQ.Z3.AND.BMAX.EQ.Z1) YPOLE=Z2
      IF(AMIN.EQ.Z3.AND.BMAX.EQ.Z2) YPOLE=Z1
      XPOLE=AMIN
      ZPOLE=BMAX
 199  H1=EM1-XPOLE
      H2=EM2-XPOLE
      H3=EM3-XPOLE
      X3=(-EM5+EM4*EM6/H2)/(H3-EM6**2/H2)
      X2=(-EM4-EM6*X3)/H2
      X1=1.0
      H4=EM1-YPOLE
      H5=EM2-YPOLE
      H6=EM3-YPOLE
      X6=(-EM5+EM4*EM6/H5)/(H6-EM6**2/H5)
      X5=(-EM4-EM6*X6)/H5
      X4=1.0
      H7=EM1-ZPOLE
      H8=EM2-ZPOLE
      H9=EM3-ZPOLE
      X9=(-EM5+EM4*EM6/H8)/(H9-EM6**2/H8)
      X8=(-EM4-EM6*X9)/H8
      X7=1.0
      S1=SQRT(X1**2+X2**2+X3**2)
      CALP1=X1/S1
      CBET1=X2/S1
      CGAM1=X3/S1
      S2=SQRT(X4**2+X5**2+X6**2)
      CALP2=X4/S2
      CBET2=X5/S2
      CGAM2=X6/S2
      S3=SQRT(X7**2+X8**2+X9**2)
      CALP3=X7/S3
      CBET3=X8/S3
      CGAM3=X9/S3
      AML1=ATAN2(CBET1,CALP1)
      AML2=ATAN2(CBET2,CALP2)
      AML3=ATAN2(CBET3,CALP3)
```

Table 7--Continued

```
      TTHA1=SQRT(1.-CGAM1**2)/CGAM1
      THA1=ATAN(ABS(TTHA1))
      IF(TTHA1) 26,27,27
   26 THA1=3.141593-THA1
   27 TTHA2=SQRT(1.-CGAM2**2)/CGAM2
      THA2=ATAN(ABS(TTHA2))
      IF(TTHA2) 28,29,29
   28 THA2=3.141593-THA2
   29 TTHA3=SQRT(1.-CGAM3**2)/CGAM3
      THA3=ATAN(ABS(TTHA3))
      IF(TTHA3) 30,31,31
   30 THA3=3.141593-THA3
   31 AML11=57.29578*AML1
      AML12=180.+AML11
      IF(AML11) 33,34,34
   33 AML11=360.+AML11
      AML12=AML11-180.
   34 AML21=57.29577*AML2
      AML22=180.+AML21
      IF(AML21) 35,36,36
   35 AML21=360.+AML21
      AML22=AML21-180.
   36 AML31=57.29578*AML3
      AML32=180.+AML31
      IF(AML31) 37,38,38
   37 AML31=360.+AML31
      AML32=AML31-180.
   38 THTA11=57.29578*THA1
      THTA12=180.-THTA11
      THTA21=57.29578*THA2
      THTA22=180.-THTA21
      THTA31=57.29578*THA3
      THTA32=180.-THTA31
      X=C22/S22
      Y=SQRT(X**2+1.)
      TP1=X+Y
      PH1=ATAN(TP1)
      SPH1=SIN(PH1)
      CPH1=COS(PH1)
      P11=(C00+.1*C20)/3.+(.1*C22*(SPH1**2-CPH1**2)+.2*S22*SPH1*CPH1)/2
      PHI1=57.29578*PH1
      TP2=X-Y
      PH2=3.141593-ATAN(ABS(TP2))
      SPH2=SIN(PH2)
      CPH2=COS(PH2)
      P22=(C00+.1*C20)/3.+(.1*C22*(SPH2**2-CPH2**2)+.2*S22*SPH2*CPH2)/2
      PHI2=57.29578*PH2
      PHMIN=AMIN1(P11,P22)
      IF(PHMIN.EQ.P11) MIN=1
      IF(PHMIN.EQ.P22) MIN=2
      WRITE(6,1010) XPOLE,AML11,THTA11,AML12,THTA12
      WRITE(6,1011) YPOLE,AML21,THTA21,AML22,THTA22
      WRITE(6,1012) ZPOLE,AML31,THTA31,AML32,THTA32
 1010 FORMAT(1H0,5H  X  ,F8.5,3X,4(3X,F8.3))
 1011 FORMAT(1H0,5H  Y  ,F8.5,3X,4(3X,F8.3))
 1012 FORMAT(1H0,5H  Z  ,F8.5,3X,4(3X,F8.3))
```

Table 7--Continued

```
      PRINT 1013
1013  FORMAT(1H0,31HMOMENT OF INERTIA IN  EAST LONG)
      PRINT 1014
1014  FORMAT(1H ,29H  EQUATORIAL PLANE        (DEG))
      GO TO (1015,1016),MIN
1015  WRITE(6,1017) P22,PHI2
1017  FORMAT(1H0,7H  MAX   ,F8.5,8X,F8.3)
      WRITE(6,1019) P11,PHI1
1019  FORMAT(1H0,7H  MIN   ,F8.5,8X,F8.3)
      RETURN
1016  WRITE(6,1017) P11,PHI1
      WRITE(6,1019) P22,PHI2
32    RETURN
      END
```

Table 7--Continued

Table 8--Symbols Used in "MAIN" Routine of Moon Modeling Program

Fortran Nomenclature	Explanation
	Variables
CNMP SNMP	$c_{n,m'}$ $s_{n,m'}$ coefficients of spherical harmonic representation of the distribution of surface features
HI	differences in elevation between maria and continents; and between circular basins and areas outside of basins
TITLE	label for identification
CNM SNM	$c_{n,m}$ $s_{n,m}$ coefficients of a surface spherical harmonic expansion of the gravity field
EN	n
CPT	$d_{n,m} = \dfrac{(2n+1)C_{n,m}}{C_{n,m'}}$
HC	$H = d_{n/m}\dfrac{1738.}{3.}$
SPT	$d_{n,m} = \dfrac{(2n+1)s_{n,m}}{s_{n,m'}}$
HS	$H = d_{n,m}\dfrac{1738.}{3.}$
	Integers
N	n
M	m
IMAX	number of n,m combinations considered
J	counter for the distributions considered (4)
K	counter for the sets of data considered (3)
L	counter for HI
LMAX	number of HI considered

Table 9--Symbols Used in "CϕMPT" subroutine of Moon Modeling Program

Fortran Nomenclature	Explanation
Variables	
A	radius of moon
R1	ρ_a, average density of moon
R2	ρ_i, density of nickel-iron
R3	ρ_{mm}, density of material beneath maria or circular basins
R4	ρ_c, density of crustal material
H	HI (see "MAIN" explanation)
T	t, thickness of nickel-iron layer
DELRA	$\Delta\rho(a)$, density contrast
X	
Y	
Z	intermediate storage locations for quadratic calculation of additional density contrasts
SQQ	
SQ	
D	depth of compensation
DRA	$\Delta\rho(a)$, density contrast
Integers	
L	counter for HI
LMAX	number of HI considered